Competitive Markets with Info-gaps

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Contents

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Abstract

We develop a theory of competitive partial equilibrium in which consumers and firms have severely deficient information about production costs. We use non-probabilistic info-gap models of uncertainty to represent this knowledge-deficiency. The analysis is based on the robustness functions for consumers and for firms, which express the degree of an agent's immunity to uncertainty of the cost functions, in decisions regarding consumption and production. Several propositions disclose trade-offs between robustness to uncertainty and the agent's aspiration for reward. Two additional propositions establish info-gap analogs of the neo-classical results which relate price to marginal cost and to marginal utility at competitive equilibrium. We discuss a Pareto-like efficiency which characterizes

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info-gap competitive equilibrium, and discuss some welfare implications. We demonstrate a "supply-side" feature of info-gap competitive equilibrium: the aspirations of firms, but not of consumers, directly influence price and aggregate consumption.

1 Introduction

In neo-classical economic theory, competitive equilibrium is a state of a market system of price-takers in which firms maximize profit, consumers maximize utility and the market is cleared. The first and most fundamental welfare theorem states that the aggregate surplus is maximized at competitive equilibrium, and this maximizes the set of possible consumer-utility values. An immediate consequence is that the market at competitive equilibrium is Pareto efficient: no consumer can enhance personal utility without diminishing the utility of some other consumer. This theory of competitive equilibrium has enjoyed some pretty harsh criticism by such diverse thinkers as Hayek [4, chap. 5], Schumpeter [10, chaps. VI and VIII] and Shackle [11, chaps. 25, 26] but it is nonetheless still a point of departure for every modern economic theory.

The particular importance of the neo-classical concept of competitive equilibrium is that it establishes a model in which individual agents (consumers and firms), when acting autonomously in the light of their personal interests, create an aggregate result which, while perhaps not fair by some standards, is at least efficient. However, Hayek has pointed out that firms are

> assumed to know the lowest cost at which the commodity can be produced. Yet this knowledge which is assumed to be given to begin with is one of the main points where it is only through the process of competition that the facts will be discovered. This appears to me one of the most important of the points where the starting-point of the theory of competitive equilibrium assumes away the main task which only the process of competition can solve. [4, pp.95–96]

There are two issues here, knowledge and time. We will deal only with the first, deferring to a later study the dynamic evolutionary nature of the competitive process. Our main result will be based on info-gap theory. We will show that even when each agent's knowledge of the cost of production is deficient, an efficient aggregate result is obtained by independently satisficing individual aspirations. We will also find a dominance of supply-side aspirations in a market at info-gap competitive equilibrium.

There has been enormous progress in the economics of deficient information since Hayek's words were written in the 1940s. However, this progress has been based almost exclusively upon probabilistic models of uncertainty. Knight repeatedly argued that the uncertainties upon which entrepreneurial competition thrives are utterly different from probabilities:

> The uncertainties which persist as causes of profit are those which are uninsurable because there is no objective measure of the probability of gain or loss. . . . Situations in regard to which business judgment must be exercised do not repeat themselves with sufficient conformity to type to make possible a computation of probability. [5, p.120]

Like Hayek, Knight is stressing knowledge and its absence. Nobody disputes that probability distributions reflect imperfect knowledge. The point is that real economic uncertainties, those which motivate the entrepreneur and are either a blessing or a bane, are starker and sparser than is reflected in frequentist or Bayesian/subjectivist measure functions. Real economic uncertainty "is the complement of knowledge. It is the gap between what is known and what needs to be known to make correct decisions." [8, p.1]. Uncertainty is an information gap: "the difference between the amount of information required to perform the task and the amount of information already possessed by the organization." [3, p.5]. In this paper we will develop an info-gap analysis of competitive equilibrium.

In section 2 we present a brief summary of info-gap models of uncertainty, upon which our analysis is based. We will use info-gap models to represent knowledge-deficiency in the cost of production. We will study Marshallian partial equilibrium, and our notation is defined in section 3.

In section 4 we introduction the basic decision functions of info-gap theory: the robustness functions for consumers and for firms. These functions express the degree of an agent's immunity to uncertainty of the cost functions, in decisions regarding consumption and production. The robustness function enables the decision maker to formulate and evaluate the feasibility of satisficing strategies. Several propositions are presented which express the trade-off between robustness to uncertainty and the agent's aspiration for reward.

Section 5 discusses marginal analysis and competitive partial equilibrium in the info-gap context. The two propositions in this section establish the info-gap analogs of the neo-classical results which relate price to marginal cost and to marginal utility at competitive equilibrium.

In section 6 we discuss the Pareto-like efficiency which characterizes info-gap competitive equilibrium, and compare this to the Pareto efficiency which underlies the basic welfare property of neo-classical competitive equilibrium. We find that a market at info-gap competitive equilibrium has no excess robustness to uncertainty: a consumer or firm can augment robustness only by reducing the aspiration for reward.

Section 7 contains an analysis of an asymmetry between the aspirations of consumers and of firms. We see a "supply-side" feature of info-gap competitive equilibrium: the aspirations of firms, but not of consumers, directly influence price and aggregate consumption.

Finally, in section 8, we illustrate the theoretical results with a simple example. Section 9 concludes our discussion. All proofs are presented in the appendix, section 10.

2 Info-gap Models of Uncertainty

Our quantification of knowledge-deficiency is based on non-probabilistic information-gap models [2]. An info-gap is a disparity between what the decision maker knows and what could be known. The range of possibilities expands as the info-gap grows. An info-gap model is a family of nested sets. Each set corresponds to a particular degree of knowledge-deficiency, according to its level of nesting. Each element in a set represents a possible event. There are no measure functions in an info-gap model.

Info-gap theory provides a quantitative model for Knight's concept of "true uncertainty" for which "there is no objective measure of the probability", as opposed to risk which is probabilistically measurable [6, pp.46, 120, 231–232]. Further discussion of the relation between Knight's conception and info-gap theory is found in [2, section 12.5]. Similarly, Shackle's "non-distributional uncertainty variable" bears some similarity to info-gap analysis [11, p.23]. Likewise, Kyburg recognized the possibility of a "decision theory that is based on some non-probabilistic measure of uncertainty." [7, p.1094].

Events are represented as vectors or vector functions c. Knowledge-deficiency is expressed at two levels by info-gap models. For fixed α the set $\mathcal{C}(\alpha,\widetilde{c})$ represents a degree of variability of c around the centerpoint $\widetilde{c}$. The greater the value of α, the greater the range of possible variation, so α is called the *uncertainty parameter* and expresses the information gap between what is known ($\widetilde{c}$ and the structure of the sets) and what needs to be known for an ideal solution (the exact value of c). The value of α is usually unknown, which constitutes the second level of imperfection of knowledge: the horizon of variation is unbounded.

Let $\Re_+$ denote the non-negative real numbers and let Ω be a Banach space in which the uncertain quantities c are defined. An info-gap model $\mathcal{C}(\alpha,\widetilde{c})$ is a map from $\Re_+ \times \Omega$ into the power set of Ω. Info-gap models obey four axioms. *Nesting:* $\mathcal{C}(\alpha,\widetilde{c}) \subseteq \mathcal{C}(\alpha',\widetilde{c})$ if $\alpha \le \alpha'$. *Contraction:* $\mathcal{C}(0,0)$ is the singleton set $\{0\}$. *Translation:* $\mathcal{C}(\alpha,\widetilde{c})$ is obtained by shifting $\mathcal{C}(\alpha,0)$ from the origin to $\widetilde{c}$: $\mathcal{C}(\alpha,\widetilde{c}) = \mathcal{C}(\alpha,0)+\widetilde{c}$. *Linear expansion:* info-gap models centered at the origin expand linearly:

$\mathcal{C}(\alpha', 0) = \frac{\alpha'}{\alpha}\mathcal{C}(\alpha, 0)$ for all α, $\alpha' > 0$. Nesting is the most characteristic of the info-gap axioms. It expresses the intuition that possibilities expand as the info-gap grows. For more discussion of these axioms see [1].

3 Marshallian Partial Equilibrium: Notation

We study Marshallian partial equilibrium, for which we use the following notation.

x_i is consumer i's consumption of good ℓ, while m_i is the monetary value of all the consumer's other consumption. Each consumer chooses $x_i \geq 0$, and m_i, the consumption of the numeraire, is a real number.

$\mathcal{I}$ denotes the set of indices of the consumers, and $\mathcal{J}$ denotes the set of indices of the firms. $\mathcal{I}$ and $\mathcal{J}$ are disjoint. $m_{\mathcal{I}}$ is the vector of m_i-values for all $i \in \mathcal{I}$, while $x_{\mathcal{I}}$ is the vector of x_i-values for all $i \in \mathcal{I}$. p is the price of good ℓ. The price of the numeraire is unity.

We assume that consumer i's utility function is quasi-linear, separable in m_i and x_i, and known:

$$u_i(m_i, x_i) = m_i + \phi_i(x_i) \tag{1}$$

ω_{mi} is consumer i's initial endowment of the numeraire. We assume that there is no initial endowment of ℓ.

θ_{ij} is consumer i's share of firm j. Consumers entirely own the firms, so $\sum_{i\in\mathcal{I}} \theta_{ij} = 1$.

q_j is the quantity of ℓ which firm j produces, and $q_{\mathcal{J}}$ is the vector of q_j-values for all $j \in \mathcal{J}$. $c_j(q_j)$ is the cost to firm j of producing q_j, in units of numeraire. That is, firm j has to consume $c_j(q_j)$ units of numeraire in order to produce q_j units of ℓ.

$\mathcal{C}_j(\alpha, \widetilde{c}_j)$, for which $\alpha \geq 0$, is the info-gap model for uncertainty in the cost function $c_j(q_j)$ for firm j. Thus $\mathcal{C}_j(\alpha, \widetilde{c}_j)$ is the set of possible cost functions, $c_j(q_j)$, for firm j, up to info-gap α. The horizon of uncertainty, α, is unknown.

4 Robustness Functions

4.1 Consumers

Consumer i's expenditures for the numeraire and for ℓ are limited by i's initial endowment and i's share in each of the firms. Hence i's budget constraint is:

$$m_i + px_i \leq \omega_{mi} + \sum_{j\in\mathcal{J}} \theta_{ij}[pq_j - c_j(q_j)] \tag{2}$$

Each cost function, $c_j(q_j)$, is uncertain and belongs to the info-gap model $\mathcal{C}_j(\alpha, \widetilde{c}_j)$. This means that no consumer (who owns shares) can know his or her budget constraint.

Consumer i aspires to consume numeraire and ℓ so as to achieve utility no less than $r_{\mathrm{c},i}$. However, i is subject to a budget constraint which is uncertain due to the unknown production-cost functions $c_j(q_j)$, $j \in \mathcal{J}$. **Consumer i's robustness** to cost uncertainty, while satisficing the utility at $r_{\mathrm{c},i}$, is, for each $i \in \mathcal{I}$:

$$\widehat{\alpha}_i(m_i, x_i, r_{\mathrm{c},i}) = \max\left\{\alpha :\ m_i + \phi_i(x_i) \geq r_{\mathrm{c},i} \text{ and } m_i + px_i \leq \omega_{mi} + \sum_{j\in\mathcal{J}} \theta_{ij}\left[pq_j - \max_{c_j\in\mathcal{C}_j(\alpha,\widetilde{c}_j)} c_j(q_j)\right]\right\} \tag{3}$$

The robustness, $\widehat{\alpha}_i(m_i, x_i, r_{\mathrm{c},i})$, is the greatest info-gap, α, at which the consumer's budget constraint is obeyed and the reward is no less than the aspiration $r_{\mathrm{c},i}$. The robustness is the maximum of a set of α-values. The constraints defining this set may force it to be empty, in which case we define $\widehat{\alpha}_i(m_i, x_i, r_{\mathrm{c},i}) = 0$.

The reward constraint in the definition of the consumer robustness function, eq.(3), may be ineffective: $m_i + \phi_i(x_i) > r_{c,i}$. Our first proposition shows that the consumer robustness function can be enhanced by satiating the reward constraint.

Definition 1 *An info-gap model is* **closed and bounded** *if all its sets are closed and bounded sets.*

Proposition 1 *The consumer's robustness can be increased by satiating the reward constraint. That is:*

Given: *a finite number of firms, closed and bounded info-gap models, values m_i' and x_i' at which the robustness is positive and the reward constraint is not satiated:*

$$\widehat{\alpha}_i(m_i', x_i', r_{c,i}) > 0 \quad \text{and} \quad m_i' + \phi_i(x_i') > r_{c,i} \tag{4}$$

Then:

$$\widehat{\alpha}_i(m_i, x_i', r_{c,i}) > \widehat{\alpha}_i(m_i', x_i', r_{c,i}) \tag{5}$$

for m_i chosen as:

$$m_i = r_{c,i} - \phi_i(x_i') \tag{6}$$

When the reward constraint is ineffective, $m_i + \phi_i(x_i) > r_{c,i}$, the robustness $\widehat{\alpha}_i(m_i, x_i, r_{c,i})$ is independent of the critical reward $r_{c,i}$. However, our next proposition shows that when $\widehat{\alpha}_i(m_i, x_i, r_{c,i})$ can be maximized on (m_i, x_i), this maximum robustness decreases monotonically with increasing $r_{c,i}$. That is, the consumer faces an irrevocable trade-off between aspiration for reward, $r_{c,i}$, and robustness against cost-uncertainty.

When the consumer's robustness can be maximized, its maximum value is denoted $\widehat{\alpha}_i(r_{c,i})$:

$$\widehat{\alpha}_i(r_{c,i}) = \max_{m_i, x_i} \widehat{\alpha}_i(m_i, x_i, r_{c,i}) \tag{7}$$

Proposition 2 Given: *the number of firms is finite and their info-gap models are closed and bounded.*

Then: *each consumer's maximal robustness decreases monotonically with increasing critical reward:*

$$r_{c,i} > r_{c,i}' \quad \text{implies} \quad \widehat{\alpha}(r_{c,i}') > \widehat{\alpha}(r_{c,i}) \tag{8}$$

if these maximal robustnesses exist.

4.2 Firms

Firm j aspires to choose a production level q_j so as to achieve profit no less than $r_{c,j}$. However, j is plagued by uncertain costs. The **robustness function for firm** j is, for each $j \in \mathcal{J}$:

$$\widehat{\alpha}_j(q_j, r_{c,j}) = \max \left\{ \alpha : \ pq_j - \max_{c_j \in \mathcal{C}_j(\alpha, \widetilde{c}_j)} c_j(q_j) \geq r_{c,j} \right\} \tag{9}$$

$\widehat{\alpha}_j(q_j, r_{c,j}) = 0$ if the set of α-values is empty. The firm's robustness, $\widehat{\alpha}_j(q_j, r_{c,j})$, is the greatest info-gap, α, at which the firm's aspiration for reward, $r_{c,j}$, is guaranteed.

Like the consumers, firms face a trade-off between robustness and aspiration, as expressed in the following two propositions.

Proposition 3 *The firm's robustness decreases monotonically with increasing critical reward at fixed production volume:*

$$r_{c,j} > r_{c,j}' \quad \text{implies} \quad \widehat{\alpha}(q_j, r_{c,j}') > \widehat{\alpha}(q_j, r_{c,j}) \tag{10}$$

if these robustnesses are positive and if the info-gap model is closed and bounded.

When the firm's robustness can be maximized, its maximum value is denoted $\widehat{\alpha}_j(r_{\mathrm{c},j})$:

$$\widehat{\alpha}_j(r_{\mathrm{c},j}) = \max_{q_j} \widehat{\alpha}_j(q_j, r_{\mathrm{c},j}) \tag{11}$$

Proposition 4 *The firm's maximal robustness decreases monotonically with increasing critical reward:*

$$r_{\mathrm{c},j} > r'_{\mathrm{c},j} \quad \text{implies} \quad \widehat{\alpha}(r'_{\mathrm{c},j}) > \widehat{\alpha}(r_{\mathrm{c},j}) \tag{12}$$

if these maximal robustnesses exist and if the info-gap model is closed and bounded.

5 Marginal Analysis and Competitive Partial Equilibrium

We are studying a market theory in which the economic agents attempt to satisfice rather than maximize utility or profit. Since these agents have highly deficient knowledge of market conditions (specifically, of production costs), they seek to cause adequate reward to be as feasible as possible. They do this by maximizing their robustness functions.

Competitive equilibrium in this info-gap context is an allocation $(m_{\mathcal{I}},\, x_{\mathcal{I}},\, q_{\mathcal{J}})$ and price p which clear the market and which bring each consumer's robustness and each firm's robustness to an unconstrained maximum. Market clearing is:

$$\sum_{i\in\mathcal{I}} x_i = \sum_{j\in\mathcal{J}} q_j \tag{13}$$

In this section we present results which establish that this info-gap version of competitive equilibrium has economic consequences quite analogous to the neo-classical theory. The two propositions in this section establish the info-gap analogs of the neo-classical results which relate price (at competitive equilibrium) to marginal cost and to marginal utility.

Proposition 5 Given: $C_j(\alpha, \widetilde{c}_j)$ *is a closed and bounded info-gap model, and the firm's robustness function* $\widehat{\alpha}_j(q_j, r_{\mathrm{c},j})$ *is positive in an open interval* Q *of* q_j*-values.*

Then, *for any* $q_j \in Q$*, the price* p *satisfies:*

$$p = \frac{\partial}{\partial q_j}\left[\max_{c_j\in\mathcal{C}_j(\widehat{\alpha}_j(q_j, r_{\mathrm{c},j}),\widetilde{c}_j)} c_j(q_j)\right] \tag{14}$$

We know from lemma 3 (in the Appendix) that the firm's reward constraint is satiated by a maximal cost function, $\max c_j(q_j)$. The differentiation with respect to q_j in proposition 5 affects both the argument of the satiating cost function $c(q_j)$ and the argument of the robustness, $\widehat{\alpha}_j(q_j, r_{\mathrm{c},j})$. When q_j is a production volume which maximizes the robustness (without constraint), then $\widehat{\alpha}_j(q_j, r_{\mathrm{c},j})$ does not vary with differential variation of q_j. In other words, proposition 5 asserts that, at maximal robustness of the firm, the price equals the marginal cost of any cost function which satiates the firm's reward constraint. This is the info-gap analog of the neo-classical result which asserts that, at competitive equilibrium, the price equals the marginal cost of production.

Proposition 6 Given: *the number of firms is finite, the info-gap models are closed and bounded, and the consumer robustness function* $\widehat{\alpha}_i(m_i, x_i, r_{\mathrm{c},i})$ *is positive for* $m_i = r_{\mathrm{c},i} - \phi_i(x_i)$ *and for all* x_i *in an open interval* X*.*

Then, *for any* $x_i \in X$*, and with* $m_i = r_{\mathrm{c},i} - \phi_i(x_i)$*, the price* p *satisfies:*

$$p = \frac{\mathrm{d}\phi_i(x_i)}{\mathrm{d}x_i} - \frac{\partial}{\partial x_i}\left[\sum_{j\in\mathcal{J}} \theta_{ij} \max_{c_j\in\mathcal{C}_j(\widehat{\alpha}_i(m_i,x_i,r_{\mathrm{c},i}),\widetilde{c}_j)} c_j(q_j)\right] \tag{15}$$

Note that the partial derivative with respect to x_i affects only the robustness, $\widehat{\alpha}_i(m_i, x_i, r_{c,i})$. If the robustness is strictly increasing as x_i increases, then the maxima in eq.(15) will all be strictly increasing in x_i as well (from the nesting axiom of info-gap models), so the partial derivative will be positive. Hence, if the robustness is strictly increasing in x_i, then the price is less than the marginal utility. The converse is also true: if the price is less than the marginal utility, then the partial derivative is positive which implies that the robustness is increasing in x_i.

Likewise, the robustness is strictly decreasing in x_i if and only if the price is greater than the marginal utility. Finally, x_i brings the robustness to an unconstrained maximum if and only if the partial derivative vanishes and price precisely equals marginal utility. In short, subject to the conditions of proposition 6:

$$\frac{\partial \widehat{\alpha}_i(m_i, x_i, r_{c,i})}{\partial x_i} \begin{array}{c} > \\ = \\ < \end{array} 0 \quad \text{if and only if} \quad p \begin{array}{c} < \\ = \\ > \end{array} \frac{\mathrm{d}\phi_i(x_i)}{\mathrm{d}x_i} \tag{16}$$

Relations (16) hold regardless of whether or not the marginal utility is increasing or decreasing. However, only if the marginal utility is a decreasing function of x_i, will the marginal utility be dynamically driven towards the price when the consumer seeks to maximize the robustness. We see this as follows.

Suppose that the marginal utility is positive and decreasing: $\mathrm{d}\phi_i(x_i)/\mathrm{d}x_i > 0$ and $\mathrm{d}^2\phi_i(x_i)/\mathrm{d}x_i^2 < 0$, as in fig. 1. At $x_{i,1}$: $\mathrm{d}\phi_i/\mathrm{d}x_i > p$ so (16) implies that $\widehat{\alpha}_i$ is increased by increasing x_i, causing $\mathrm{d}\phi_i/\mathrm{d}x_i$ to approach p. The same convergence of $\mathrm{d}\phi_i/\mathrm{d}x_i$ to p occurs at $x_{i,2}$.

However, if the marginal utility is increasing, $\mathrm{d}^2\phi_i(x_i)/\mathrm{d}x_i^2 > 0$, as in fig. 2, then the robustness at $x_{i,1}$ is increased by decreasing x_i and thus moving $\mathrm{d}\phi_i/\mathrm{d}x_i$ away from p. The same divergence of $\mathrm{d}\phi_i/\mathrm{d}x_i$ from p occurs at $x_{i,2}$.

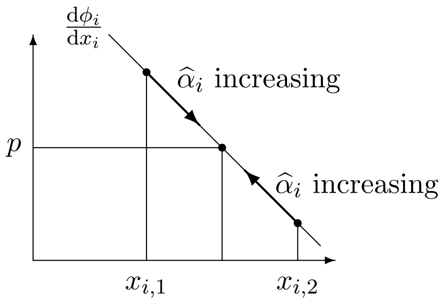

Figure 1: Decreasing marginal utility: convergence towards the price.

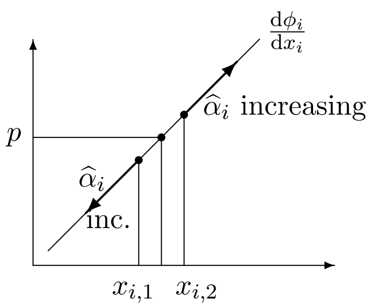

Figure 2: Increasing marginal utility: divergence from the price.

6 Welfare

The info-gap concept of competitive equilibrium discussed in section 5 entails a Pareto-like efficiency, which we present here.

Neo-classical competitive equilibrium involves maximization of utility by consumers and of profit by firms. The fundamental neo-classical welfare theorem states that competitive equilibrium is Pareto efficient in the sense that the aggregate utility is maximized with respect to specified market conditions. The utility of any consumer can be increased only at the expense of some other consumer [9, section 10.D]. There is no wasted utility lurking around and waiting to be exploited in a market at neo-classical competitive equilibrium.

In info-gap competitive equilibrium the agents satisfice with respect to their aspirations for utility or profit, and maximize their robustness. That is, robustness is maximized (at fixed aspirations), rather than utility or profit. To explore the Pareto efficiency of a market at info-gap competitive equilibrium, we must ask if there is excess robustness which could be obtained. Propositions 2 and 4 provide an answer.

Proposition 2 asserts that, if consumer i's robustness is maximized with utility-aspiration $r_{\mathrm{c},i}$, then any increase in i's aspiration will necessarily reduce i's robustness. In other words, at info-gap competitive equilibrium, i's consumption is the most robust which is possible given i's aspiration for utility. Each consumer can achieve greater robustness only by forgoing some aspired reward.

Proposition 4 makes the analogous assertion regarding each firm: at info-gap competitive equilibrium no firm can improve its robustness without relinquishing some aspiration for profit.

In short, propositions 2 and 4 show that info-gap competitive equilibrium is efficient with respect to robustness, at fixed aspirations, individually for each consumer and for each firm. There is no wasted robustness lurking around and waiting to be exploited in a market at info-gap competitive equilibrium. One can think of robustness as a measure of the feasibility of an agent's aspiration: great robustness means that the aspiration will be achieved even if the unknown production costs vary greatly from their nominal values; low robustness implies great vulnerability to cost fluctuation. The agents in a market at info-gap competitive equilibrium have adopted actions which are maximally feasible with respect to their aspirations.

7 Aspirations and Supply-side Economics

In this section we briefly examine an asymmetry between the aspirations of consumers and of firms. We will explain that, at info-gap competitive equilibrium, consumer aspirations do not directly influence the level of consumption or the price, while the aspirations of firms directly affect both the level of production and the price.

We showed, in discussing proposition 6 and eq.(16), that price equals marginal utility when the consumer's robustness is at an unconstrained maximum: $p = \mathrm{d}\phi_i(x_i)/\mathrm{d}x_i$. In other words, at info-gap competitive equilibrium, if the utility from consumption, $\phi_i(x_i)$, does not depend upon the consumer's aspiration for reward, $r_{\mathrm{c},i}$, then the robust-optimal consumption $\widehat{x}_i$ (which maximizes $\widehat{\alpha}_i$), is independent of $r_{\mathrm{c},i}$. At competitive equilibrium, each consumer's consumption depends upon the consumer's marginal utility $\mathrm{d}\phi_i/\mathrm{d}x_i$, but not on his or her aspiration for utility $r_{\mathrm{c},i}$. The consumer's robustness at competitive equilibrium, $\widehat{\alpha}_i(r_{\mathrm{c},i})$, tells the consumer whether or not the aspiration $r_{\mathrm{c},i}$ is robust (and hence feasible) or not. If the robustness is low the consumer may decide to leave the market or may revise the aspiration to a more realistic level. Leaving the market most certainly influences the level of consumption, but this is not part of our model of competitive equilibrium, which assumes market clearing. If the consumer revises his or her aspiration to a lower level, thereby enhancing the robustness, the consumption at competitive equilibrium does not change. In short, consumer aspirations do not directly influence price or consumption at info-gap competitive equilibrium.

The situation of firms is different, and proposition 5 implies no such removal of the firm's aspiration for profit, $r_{c,j}$, from the condition for info-gap competitive equilibrium. On the contrary, in discussing proposition 5 we showed that, at competitive equilibrium, the price equals the marginal cost of a cost function which satiates the firm's reward constraint.

It will usually be the case that a firm's optimal production volume $\widehat{q}_j$ (which maximizes the firm's robustness) will increase with increasing aspiration:

$$\frac{\mathrm{d}\widehat{q}_j(r_{c,j})}{\mathrm{d}r_{c,j}} > 0 \tag{17}$$

When this holds we will say that the optimal production volume is **reward coherent** [2, p.120]. Reward coherence means that the firm must produce more if it wishes to satisfice at a higher level of aspiration for profit.

Since the market clears at competitive equilibrium, greater production means that at least some consumers must consume more. We showed in connection with figs. 1 and 2 that price converges towards marginal utility only if marginal utility is decreasing. Hence, greater consumption (to clear the market) implies lower price and lower marginal utility. Increasing aspirations of firms result in price reduction, enhanced consumption, and reduced marginal utility of consumers.

8 Example

We will illustrate our results with a simple example. Let us suppose that production has no sunk cost, and that firms and consumers do not know the rate of variation of the disparity, $c(q_j) - \widetilde{c}(q_j)$, between the actual and the nominal cost functions, $c(q_j)$ and $\widetilde{c}(q_j)$ respectively. A typical info-gap model for this situation is the following family of nested sets:

$$\mathcal{C}(\alpha, \widetilde{c}) = \left\{ c(q_j) : \ c(0) = 0, \ \left| \frac{\mathrm{d}[c(q_j) - \widetilde{c}(q_j)]}{\mathrm{d}q_j} \right| \le \alpha \right\}, \quad \alpha \ge 0 \tag{18}$$

For fixed α, $\mathcal{C}(\alpha, \widetilde{c})$ is the set of cost functions $c(q_j)$ whose rate of deviation from the known, nominal cost function $\widetilde{c}(q_j)$, is unknown but bounded. The horizon of uncertainty, α, which bounds the rate of variation of the cost function, is unknown, so the info-gap model is a family of nested sets of possible cost functions. We will adopt eq.(18) as the consumers' and firms' model of cost-uncertainty in this example.

Let the nominal cost function be:

$$\widetilde{c}(q_j) = c_0 q_j^{1+\eta} \tag{19}$$

where $c_0 > 0$ and η is a real number. The nominal marginal cost of production, $\mathrm{d}\widetilde{c}(q_j)/\mathrm{d}q_j$, is positive and increasing in q_j if $\eta > 0$.

The firm's robustness function, eq.(9), is found to be:

$$\widehat{\alpha}_j(q_j, r_{c,j}) = p - \frac{r_{c,j}}{q_j} - c_0 q_j^{\eta} \tag{20}$$

unless this expression is negative, in which case $\widehat{\alpha}_j(q_j, r_{c,j}) = 0$. The robustness, if it is positive, is strictly decreasing as the firm's aspiration, $r_{c,j}$, increases, as expected from proposition 3.

The firm's robustness function has a maximum if $\eta > 0$. This robust-optimal production volume is:

$$\widehat{q}_j(r_{c,j}) = \left(\frac{r_{c,j}}{c_0 \eta} \right)^{\frac{1}{1+\eta}} \tag{21}$$

We note that the optimal production volume is reward-coherent, eq.(17): greater aspiration, $r_{c,j}$, calls for greater production, $\widehat{q}_j(r_{c,j})$. The optimal robustness, $\widehat{\alpha}_j(\widehat{q}_j, r_{c,j})$, is strictly decreasing as $r_{c,j}$ increases, as expected from proposition 4. Furthermore, proposition 5 is also satisfied.

Now let us assume that there is only a single firm. Consumer i's robustness function, eq.(3), based on the cost-function info-gap model of eq.(18), is:

$$\widehat{\alpha}_i(m_i, x_i, r_{\mathrm{c},i}) = p - \frac{m_i + px_i - \omega_{mi}}{q_j} - c_0 q_j^{\eta} \tag{22}$$

Choosing $m_i = r_{\mathrm{c},i} - \phi_i(x_i)$ to satiate the reward constraint, we find the variation of the robustness with x_i is:

$$\frac{\partial \widehat{\alpha}_i}{\partial x_i} = \frac{1}{q_j}\left(\frac{\mathrm{d}\phi_i(x_i)}{\mathrm{d}x_i} - p\right) \tag{23}$$

which concurs with eq.(16), obtained from proposition 6. The robustness has a maximum (rather than a minimum) if the marginal utility is a decreasing function, and this maximum occurs when the marginal utility equals the price. In other words, as explained in connection with figs. 1 and 2, the marginal utility will converge towards the price only if the marginal utility is a decreasing function.

The robust-optimal level of consumption, $\widehat{x}_i$, is obtained by causing the derivative in eq.(23) to vanish. $\widehat{x}_i$ does not depend on the consumer's aspiration if the utility function, $\phi_i(x_i)$, is independent of the consumer's aspiration, $r_{\mathrm{c},i}$, as noted in section 7.

It is instructive to see how the market-clearing condition determines the price. Consider an arbitrary number of consumers whose utility functions are:

$$\phi_i(x_i) = \mu_i x_i^{1/2}, \quad i \in \mathcal{I} \tag{24}$$

where the coefficients μ_i are positive. The robust-optimal consumption for consumer i is, from (23):

$$\widehat{x}_i = \left(\frac{\mu_i}{2p}\right)^2 \tag{25}$$

For a single firm, the market-clearing condition at info-gap competitive equilibrium is:

$$\widehat{q}_j = \sum_{i \in \mathcal{I}} \widehat{x}_i \tag{26}$$

Employing eqs.(21) and (25) and solving for the price one finds:

$$p = \sqrt{\left(\frac{c_0 \eta}{r_{\mathrm{c},j}}\right)^{\frac{1}{1+\eta}} \sum_{i \in \mathcal{I}} \frac{\mu_i^2}{4}} \tag{27}$$

We note that the price at competitive equilibrium falls as the firm's aspiration for profit, $r_{\mathrm{c},j}$, rises, as anticipated in the discussion at the end of section 7. The price at equilibrium is independent of consumer aspirations $r_{\mathrm{c},i}$.

9 Summary and Conclusion

Competition is a market mechanism with distinctive economic consequences, most notably, Pareto efficiency and the role of price in guiding agents towards actions which match aspirations. Meaningful economic analysis can be performed by stripping away the dynamic evolutionary aspect of the competitive process, and studying stationary competitive equilibrium. However, this extraction of competition from its natural dynamic environment does not require an equally violent assumption of full information. The distinctive contribution of the current paper is to demonstrate an analysis of competitive equilibrium in which the agents have severely deficient knowledge of the costs of production. We use info-gap models of uncertainty, which are less structured and less informative than probabilistic models. We consider agents whose decisions are based on satisficing their aspirations, and who choose their actions to be as robust as possible to their own information gaps.

A market at info-gap competitive equilibrium is one in which the market clears, consumers and firms maximize their robustness to uncertainty (rather than their utility or their profit) and satisfice their utility or profit. Our main results, in section 5, demonstrate the relation between price, marginal cost and marginal utility, in a market at info-gap competitive equilibrium. These results, propositions 5 and 6, are analogous to, though distinct from, the neo-classical marginal analysis of competitive equilibrium.

Info-gap robustness functions, upon which our analysis depends, display an irrevocable trade-off between robustness to uncertainty and aspiration for reward. Proposition 2 in section 4 shows that a consumer can enhance his or her robustness to knowledge-deficiency only by relinquishing aspiration for utility. Similarly, propositions 3 and 4 show that firms can augment robustness only by forgoing aspiration for profit. These results underlie the Pareto-like efficiency of a market at info-gap competitive equilibrium, discussed in section 6: there is no extra, unutilized, robustness which can be exploited without reducing the aspirations of agents in the market.

Finally, we have shown that info-gap competitive equilibrium has a distinctively supply-side bias. In section 7 we found, by assuming market-clearing, that price and production level are independent of consumer aspirations for utility, but are directly influenced by the profit-aspirations of firms. In part, this may be a result of the fact that we have considered uncertainty only in production costs, and ignored uncertainty in consumer utility functions.

Much remains to be explored, in addition to restoring competition to its temporal dynamic environment. Economic agents have knowledge deficiencies in areas other than production costs, such as consumer utility, government policy, future technological or commodity innovations, etc. In addition, robust satisficing, which underlies the present analysis, is not the only available model of decision-making with severe info-gaps. The search for robustness is a response to the pernitious potential of the unknown. However, uncertainty may be propitious, and can induce decision makers to pursue opportune windfalling strategies, as discussed in [2]. Such strategies can interact in various ways with robust satisficing in a competitive environment.

10 Appendix: Proofs

Let $\Re$ denote the real numbers.

10.1 Proofs for section 4

Lemma 1 If $\mathcal{C}(\alpha,\widetilde{c})$ *is a closed and bounded info-gap model of real-valued functions* $c(q)$, **then,** *for all* $\alpha > 0$ *and all* $\delta > 0$*:*

$$\max_{c\in\mathcal{C}(\alpha+\delta,\widetilde{c})} c(q) = \max_{c\in\mathcal{C}(\alpha,\widetilde{c})} c(q) + \frac{\delta}{\alpha}\max_{c\in\mathcal{C}(\alpha,0)} c(q) \tag{28}$$

Proof of lemma 1. The following relations result from the translation and expansion axioms of info-gap models, for all $\alpha > 0$ and all $\delta > 0$:

$$\mathcal{C}(\alpha,\widetilde{c}) = \mathcal{C}(\alpha,0)+\widetilde{c} \tag{29}$$

$$\mathcal{C}(\alpha+\delta,\widetilde{c}) = \frac{\alpha+\delta}{\alpha}\mathcal{C}(\alpha,0)+\widetilde{c} \tag{30}$$

Consequently, since the info-gap models are closed and bounded and the functions are real-valued, for any fixed value of q we have:

$$\max_{c\in\mathcal{C}(\alpha,\widetilde{c})} c(q) = \max_{c\in\mathcal{C}(\alpha,0)} c(q)+\widetilde{c}(q) \tag{31}$$

$$\max_{c\in\mathcal{C}(\alpha+\delta,\widetilde{c})} c(q) \quad = \quad \frac{\alpha+\delta}{\alpha} \max_{c\in\mathcal{C}(\alpha,0)} c(q) + \widetilde{c}(q) \tag{32}$$

The last two relations yield the desired result. ∎

Lemma 2 Given: *the number of firms is finite, the info-gap models are closed and bounded, and consumer i's robustness function is positive. Denote* $\widehat{\alpha}_i = \widehat{\alpha}_i(m_i, x_i, r_{c,i})$.

Then: *consumer i's budget constraint is satiated for some selection of cost functions* $c_j \in \mathcal{C}_j(\widehat{\alpha}_i, \widetilde{c}_j)$, *for all* $j \in \mathcal{J}$. *That is,*

$$m_i + px_i = \omega_{mi} + \sum_{j\in\mathcal{J}} \theta_{ij} \left[pq_j - \max_{c_j\in\mathcal{C}_j(\widehat{\alpha}_i,\widetilde{c}_j)} c_j(q_j) \right] \tag{33}$$

Furthermore, if m_i' *and* x_i' *maximize* $\widehat{\alpha}_i(m_i, x_i, r_{c,i})$ *on* $m_i \in \Re$ *and* $x_i \geq 0$, *then the reward constraint is also satiated:*

$$m_i' + \phi_i(x_i') = r_{c,i} \tag{34}$$

Proof of lemma 2. We first prove the assertion of budget satiation, relation (33), then we prove reward satiation, relation (34).

1. Budget constraint. Suppose, in contradiction to the proposition, that the budget constraint in the consumer robustness function, eq.(3), were a strict inequality, and let $B > 0$ be the budget excess. That is:

$$m_i + px_i = -B + \omega_{mi} + \sum_{j\in\mathcal{J}} \theta_{ij} \left[pq_j - \max_{c_j\in\mathcal{C}_j(\widehat{\alpha}_i,\widetilde{c}_j)} c_j(q_j) \right] \tag{35}$$

From lemma 1 this we conclude that, for $\alpha > 0$ and $\delta > 0$:

$$\max_{c\in\mathcal{C}(\alpha+\delta,\widetilde{c})} c(q) = \max_{c\in\mathcal{C}(\alpha,\widetilde{c})} c(q) + \frac{\delta}{\alpha} \max_{c\in\mathcal{C}(\alpha,0)} c(q) \tag{36}$$

Since $\widehat{\alpha}_i > 0$, this implies that, for each $j \in \mathcal{J}$, we can choose a $\delta_j > 0$ such that:

$$\max_{c_j\in\mathcal{C}_j(\widehat{\alpha}_i+\delta_j,\widetilde{c}_j)} c_j(q_j) \leq B + \max_{c_j\in\mathcal{C}_j(\widehat{\alpha}_i,\widetilde{c}_j)} c_j(q_j) \tag{37}$$

Multiplying by θ_{ij} and summing on j, relation (37) can be re-written:

$$\sum_{j\in\mathcal{J}} \theta_{ij} \left[B + \max_{c_j\in\mathcal{C}_j(\widehat{\alpha}_i,\widetilde{c}_j)} c_j(q_j) \right] \geq \sum_{j\in\mathcal{J}} \theta_{ij} \max_{c_j\in\mathcal{C}_j(\widehat{\alpha}_i+\delta_j,\widetilde{c}_j)} c_j(q_j) \tag{38}$$

Define:

$$\delta = \min_{j\in\mathcal{J}} \delta_j \tag{39}$$

which is positive since all the δ_j are positive and $\mathcal{J}$ is finite. From the nesting axiom of info-gap models, and recalling that $\sum_{j\in\mathcal{J}} \theta_{ij} = 1$, we obtain:

$$B + \sum_{j\in\mathcal{J}} \theta_{ij} \max_{c_j\in\mathcal{C}_j(\widehat{\alpha}_i,\widetilde{c}_j)} c_j(q_j) \geq \sum_{j\in\mathcal{J}} \theta_{ij} \max_{c_j\in\mathcal{C}_j(\widehat{\alpha}_i+\delta,\widetilde{c}_j)} c_j(q_j) \tag{40}$$

Subtracting (40) from (35) implies:

$$m_i + px_i \leq \omega_{mi} + \sum_{j\in\mathcal{J}} \theta_{ij} \left[pq_j - \max_{c_j\in\mathcal{C}_J(\widehat{\alpha}_i+\delta,\widetilde{c}_j)} c_j(q_j) \right] \tag{41}$$

Therefore the budget constraint is observed up to info-gap $\widehat{\alpha}_i + \delta$. We conclude that the robustness is no less than $\widehat{\alpha}_i + \delta$:

$$\widehat{\alpha}_i(m_i, x_i, q_{\mathcal{J}}, r_{\mathrm{c},i}, p) \geq \widehat{\alpha}_i(m_i, x_i, q_{\mathcal{J}}, r_{\mathrm{c},i}, p) + \delta \tag{42}$$

which is a contradiction, so there can be no budget excess: $B = 0$.

2. Reward constraint. Denote the maximized robustness by $\widehat{\alpha}_i = \widehat{\alpha}_i(m_i', x_i', r_{\mathrm{c},i})$. We have already shown that the budget constraint is satiated with m_i' and x_i'. Concerning the reward constraint, suppose that:

$$m_i' + \phi_i(x_i') > r_{\mathrm{c},i} \tag{43}$$

Then there is a $B > 0$ such that $m_i'' = m_i' - B$ and:

$$m_i'' + \phi_i(x_i') = r_{\mathrm{c},i} \tag{44}$$

Hence the satiated budget constraint for robustness $\widehat{\alpha}_i$ becomes:

$$m_i'' + px_i' = -B + \omega_{mi} + \sum_{j \in \mathcal{J}} \theta_{ij} \left[pq_j - \max_{c_j \in \mathcal{C}_J(\widehat{\alpha}_i, \widetilde{c}_j)} c_j(q_j) \right] \tag{45}$$

For this value of B, we can choose $\delta_j > 0$ for each $j \in \mathcal{J}$ so as to satisfy relation (37). Hence eqs.(38)–(39) hold, leading to the analog of (41) for some $\delta > 0$:

$$m_i'' + px_i' \leq \omega_{mi} + \sum_{j \in \mathcal{J}} \theta_{ij} \left[pq_j - \max_{c_j \in \mathcal{C}_J(\widehat{\alpha}_i + \delta, \widetilde{c}_j)} c_j(q_j) \right] \tag{46}$$

This results in the conclusion, as in eq.(42), that $\widehat{\alpha}_i(m_i'', x_i', q_{\mathcal{J}}, r_{\mathrm{c},i}, p) \geq \widehat{\alpha}_i(m_i', x_i', q_{\mathcal{J}}, r_{\mathrm{c},i}, p) + \delta$ which is a contradiction since m_i', x_i' maximize $\widehat{\alpha}_i(\ldots)$. Hence the supposition in (43) is false and the reward constraint in an equality. ∎

Proof of proposition 1. For convenience we will denote $\widehat{\alpha}_i' = \widehat{\alpha}_i(m_i', x_i', r_{\mathrm{c},i})$.

Since $m_i' + \phi_i(x_i') > r_{\mathrm{c},i}$, choose $B > 0$ such that $m_i = m_i' - B$ and $m_i + \phi_i(x_i') = r_{\mathrm{c},i}$. By lemma 2, the budget constraint is satiated at m_i' and x_i' since $\widehat{\alpha}_i' > 0$:

$$m_i' + px_i' = \omega_{mi} + \sum_{j \in \mathcal{J}} \theta_{ij} \left[pq_j - \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_i', \widetilde{c}_j)} c_j(q_j) \right] \tag{47}$$

Thus:

$$m_i + px_i' = -B + \omega_{mi} + \sum_{j \in \mathcal{J}} \theta_{ij} \left[pq_j - \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_i', \widetilde{c}_j)} c_j(q_j) \right] \tag{48}$$

From lemma 1 and since $\widehat{\alpha}_i' > 0$, we conclude that, for any $\delta_j > 0$:

$$\max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_i' + \delta_j, \widetilde{c}_j)} c_j(q_j) = \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_i', \widetilde{c}_j)} c_j(q_j) + \frac{\delta_j}{\widehat{\alpha}_i'} \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_i', 0)} c_j(q_j) \tag{49}$$

Hence we can choose $\delta_j > 0$ such that:

$$\max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_i' + \delta_j, \widetilde{c}_j)} c_j(q_j) \leq B + \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_i', \widetilde{c}_j)} c_j(q_j) \tag{50}$$

Multiply (50) by θ_{ij} and sum on $j \in \mathcal{J}$:

$$\sum_{j \in \mathcal{J}} \theta_{ij} \left[B + \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_i', \widetilde{c}_j)} c_j(q_j) \right] \geq \sum_{j \in \mathcal{J}} \theta_{ij} \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_i' + \delta_j, \widetilde{c}_j)} c_j(q_j) \tag{51}$$

Define:

$$\delta = \min_{j \in \mathcal{J}} \delta_j \tag{52}$$

which is positive since $\mathcal{J}$ is a finite set. From the nesting axiom of info-gap models and since $\sum_{j \in \mathcal{J}} \theta_{ij} = 1$, relation (51) becomes:

$$B + \sum_{j \in \mathcal{J}} \theta_{ij} \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}'_i, \widetilde{c}_j)} c_j(q_j) \geq \sum_{j \in \mathcal{J}} \theta_{ij} \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}'_i + \delta, \widetilde{c}_j)} c_j(q_j) \tag{53}$$

Subtracting (53) from (48) leads to:

$$m_i + p x'_i \leq \omega_{mi} + \sum_{j \in \mathcal{J}} \theta_{ij} \left[p q_j - \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}'_i + \delta, \widetilde{c}_j)} c_j(q_j) \right] \tag{54}$$

Thus the budget constraint with m_i and x'_i is satisfied up to $\alpha = \widehat{\alpha}'_i + \delta$. Hence $\widehat{\alpha}_i(m_i, x'_i, r_{c,i}) \geq \widehat{\alpha}'_i + \delta > \widehat{\alpha}'_i$. ∎

Proof of proposition 2. Let (m_i, x_i) be a consumption pair which maximizes $\widehat{\alpha}_i(m_i, x_i, r_{c,i})$. From lemma 2, (m_i, x_i) satiates the reward and robustness constraints. Hence, since $r_{c,i} > r'_{c,i}$, $\widehat{\alpha}_i(m_i, x_i, r'_{c,i})$ exists and is no less than $\widehat{\alpha}_i(m_i, x_i, r_{c,i})$. That is: $\widehat{\alpha}_i(m_i, x_i, r'_{c,i}) \geq \widehat{\alpha}_i(m_i, x_i, r_{c,i})$. Finally, since $m_i + \phi_i(x_i) = r_{c,i} > r'_{c,i}$, proposition 1 implies that $\widehat{\alpha}_i(m_i, x_i, r'_{c,i}) < \widehat{\alpha}_i(r'_{c,i})$. ∎

Lemma 3 Given: *$\mathcal{C}_j(\alpha, \widetilde{c}_j)$ is a closed and bounded info-gap model and firm j's robustness function $\widehat{\alpha}_j(q_j, r_{c,j})$ is positive.*

Then *the reward constraint is satiated for some cost function $c_j \in \mathcal{C}_j(\widehat{\alpha}_j(q_j, r_{c,j}), \widetilde{c}_j)$. That is, defining $\widehat{\alpha}_j = \widehat{\alpha}_j(q_j, r_{c,j})$:*

$$p q_j - \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_j, \widetilde{c}_j)} c_j(q_j) = r_{c,j} \tag{55}$$

Proof of lemma 3. Suppose the firm's reward constraint is a strict inequality for all $c_j \in \mathcal{C}_j(\widehat{\alpha}_j, \widetilde{c}_j)$, so there is an $R > 0$ such that:

$$p q_j - \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_j, \widetilde{c}_j)} c_j(q_j) = r_{c,j} + R \tag{56}$$

Then, from lemma 1, we can choose $\delta > 0$ such that:

$$\max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_j + \delta, \widetilde{c}_j)} c_j(q_j) \leq R + \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_j, \widetilde{c}_j)} c_j(q_j) \tag{57}$$

Hence, subtracting (57) from (56) results in:

$$p q_j - \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_j + \delta, \widetilde{c}_j)} c_j(q_j) \geq r_{c,j} \tag{58}$$

so $\widehat{\alpha}_j \geq \widehat{\alpha}_j + \delta$ which is a contradiction. Hence the supposition in (56) is false. ∎

Proof of proposition 3. Let $\widehat{\alpha}_j = \widehat{\alpha}_j(q_j, r_{c,j})$. From lemma 3, the reward constraint is satiated:

$$p q_j - \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_j, \widetilde{c}_j)} c_j(q_j) = r_{c,j} \tag{59}$$

From lemma 1 we can choose δ small enough so that:

$$\max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_j + \delta, \widetilde{c}_j)} c_j(q_j) < \max_{c_j \in \mathcal{C}(\widehat{\alpha}_j, \widetilde{c}_j)} c_j(q_j) + r_{c,j} - r'_{c,j} \tag{60}$$

Subtracting (60) from (59):

$$p q_j - \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_j + \delta, \widetilde{c}_j)} c_j(q_j) > r'_{c,j} \tag{61}$$

Hence $\widehat{\alpha}_j(q_j, r'_{c,j}) \ge \widehat{\alpha}_j + \delta > \widehat{\alpha}_j$. ■

Proof of proposition 4. Let q_j and q'_j be production volumes which maximize the firm's robustness at $r_{c,j}$ and at $r'_{c,j}$, respectively. Since $r_{c,j} > r'_{c,j}$, proposition 3 asserts:

$$\widehat{\alpha}_j(q_j, r'_{c,j}) > \widehat{\alpha}_j(q_j, r_{c,j}) = \widehat{\alpha}_j(r_{c,j}) \tag{62}$$

By definition:

$$\widehat{\alpha}_j(r'_{c,j}) \ge \widehat{\alpha}_j(q_j, r'_{c,j}) \tag{63}$$

which completes the proof. ■

10.2 Proofs for section 5

Proof of proposition 5. Choose $q_j \in Q$. Since Q is an open interval, there is a positive ε_0 such that $q_j + \varepsilon \in Q$ for all positive ε less than ε_0. Thus, by the supposition of the proposition, the robustnesses $\widehat{\alpha}_j(q_j, r_{c,j})$ and $\widehat{\alpha}_j(q_j + \varepsilon, r_{c,j})$ are both positive. By lemma 3, the reward constraints for both robustnesses are satiated:

$$\begin{aligned} r_{c,j} &= pq_j - \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_j(q_j, r_{c,j}), \widetilde{c}_j)} c_j(q_j) && (64) \\ &= p(q_j + \varepsilon) - \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_j(q_j+\varepsilon, r_{c,j}), \widetilde{c}_j)} c_j(q_j + \varepsilon) && (65) \end{aligned}$$

Subtracting (65) from (64) and re-arranging:

$$p = \frac{1}{\varepsilon}\left[\max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_j(q_j+\varepsilon, r_{c,j}), \widetilde{c}_j)} c_j(q_j + \varepsilon) - \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_j(q_j, r_{c,j}), \widetilde{c}_j)} c_j(q_j)\right] \tag{66}$$

which holds for all ε in $(0, \varepsilon_0)$. Thus the derivative in eq.(14) exists and equals p. ■

Proof of proposition 6. Choose $x_i \in X$. Since X is an open interval, there is a positive ε_0 such that $x_i + \varepsilon \in X$ for all positive ε less than ε_0. Thus, by the supposition of the proposition, and with $m_i = r_{c,i} - \phi_i(x_i)$ and with $m'_i = r_{c,i} - \phi_i(x_i + \varepsilon)$, the robustnesses $\widehat{\alpha}_i(m_i, x_i, r_{c,i})$ and $\widehat{\alpha}_j(m'_i, x_i + \varepsilon, r_{c,i})$ are both positive.

By lemma 2 the consumer's budget constraint with the choice of m_i and x_i is satiated and becomes:

$$r_{c,i} + px_i - \phi_i(x_i) = \omega_{mi} + \sum_{j \in \mathcal{J}} \theta_{ij}\left[pq_j - \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_i(m_i, x_i, r_{c,i}), \widetilde{c}_j)} c_j(q_j)\right] \tag{67}$$

Likewise, the satiated budget constraint with $x_i + \varepsilon$ and m'_i is:

$$r_{c,i} + p(x_i + \varepsilon) - \phi_i(x_i + \varepsilon) = \omega_{mi} + \sum_{j \in \mathcal{J}} \theta_{ij}\left[pq_j - \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_i(m'_i, x_i+\varepsilon, r_{c,i}), \widetilde{c}_j)} c_j(q_j)\right] \tag{68}$$

Subtracting (68) from (67) and re-arranging leads to:

$$p = \frac{1}{\varepsilon}\left[\phi_i(x_i + \varepsilon) - \phi_i(x_i)\right] - \frac{1}{\varepsilon}\sum_{j \in \mathcal{J}} \theta_{ij}\left[\max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_i(m'_i, x_i+\varepsilon, r_{c,i}), \widetilde{c}_j)} c_j(q_j) - \max_{c_j \in \mathcal{C}_j(\widehat{\alpha}_i(m_i, x_i, r_{c,i}), \widetilde{c}_j)} c_j(q_j)\right] \tag{69}$$

which holds for all ε in $(0, \varepsilon_0)$. Thus the derivatives in (15) exist and their difference equals p. ■

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