

# Project Management with Uncertain Task Durations

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**Verbal description.** A project includes  $N$  tasks, all of which must be completed. The tasks are organized in a flow chart (e.g. fig. 1) which determines the sequence in which the tasks are implemented. The flow chart is uni-directional, with branching and converging paths. The durations of these tasks are estimated but at least some durations are highly uncertain. The manager can modify the flow chart, or invest resources to change the estimated task times and their uncertainties. **Questions:** What is a reliable estimate of the overall project duration? Which task-paths are particularly vulnerable to uncertainty and require special attention? How should resources be allocated between tasks? We will outline partial answers to these questions. Much more extensive analysis is available in the sources cited below.

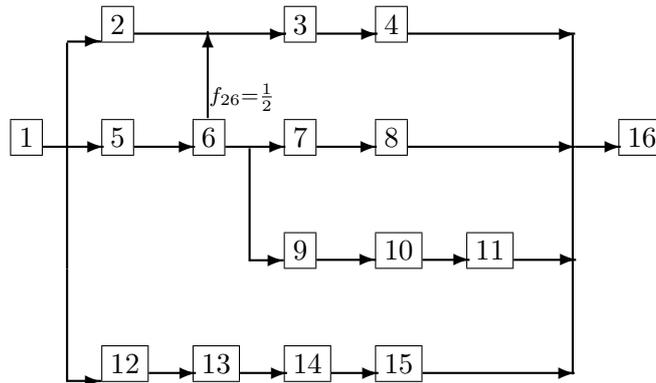


Figure 1: A 16-task project schedule.

As an example, consider the 16-task project flow chart in fig. 1. Estimated durations,  $\tilde{t}_i$ , and errors of these estimates,  $w_i$ , are given in table 1. These estimated durations are highly uncertain, and the actual values may deviate greatly from these values. In fact, for each task, the fractional deviation of the true duration,  $t_i$ , from the estimated value,  $\tilde{t}_i$ , in units of the estimated error,  $w_i$ , is unknown:

$$\left| \frac{t_i - \tilde{t}_i}{\tilde{t}_i} \right| \leq w_i h, \quad h \geq 0 \quad (1)$$

Furthermore, the horizon of uncertainty  $h$  is also unknown, so there is no known worst case.

$i$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\tilde{t}_i$	1	4	2	2	3	6	3	5	4	1	4	2	3	1	1	2
$w_i$	1	2	1	1	2	2	2	2	0.5	0.5	0.5	1	1	1	1	1

Table 1: Nominal durations and uncertainty weights.

We require that the total project duration,  $T(t)$ , not exceed a critical value,  $T_c$ :

$$T(t) \leq T_c \quad (2)$$

Since the vector of task-times,  $t$ , is highly uncertain, we cannot know if the project will complete in time. We are, however, able to determine how robust to uncertainty we are.

The **robustness** is the greatest value of the horizon of uncertainty,  $h$ , up to which the project is guaranteed to satisfy the requirement in eq.(2). The evaluation of the robustness is explained in

<sup>0</sup>\website\igt\simple-examples\uncer-task-times01.tex 26.3.2009 ©Yakov Ben-Haim.

the sources cited below. Basically, we evaluate the robustness of each path of tasks. The overall robustness of the project equals the robustness of the most vulnerable path. Let  $\hat{h}_m$  denote the robustness of the  $m$ th path, and let  $\hat{h}$  denote the overall project robustness. Then, since all task-paths must be completed, we understand that:

$$\hat{h} = \min_m \hat{h}_m \quad (3)$$

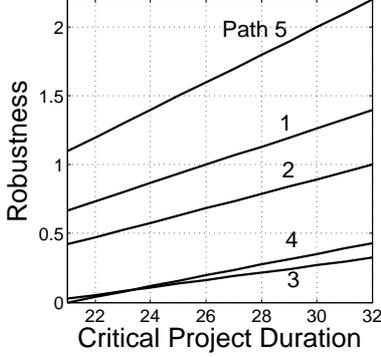


Figure 2: Robustnesses of the 5 task-paths.

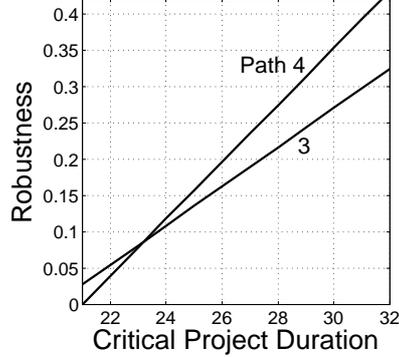


Figure 3: Robustnesses of paths 3 and 4.

The 16-task project shown in the flow chart of fig. 1 has 5 task-paths. That is, there are 5 different (though partially overlapping) sequences of tasks from task 1 to task 16:

Path 1:  $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 16$ .

Path 2:  $1 \rightarrow 5 \rightarrow 6 \rightarrow 3 \rightarrow 4 \rightarrow 16$ .

Path 3:  $1 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 16$ .

Path 4:  $1 \rightarrow 5 \rightarrow 6 \rightarrow 9 \rightarrow 10 \rightarrow 11 \rightarrow 16$ .

Path 5:  $1 \rightarrow 12 \rightarrow 13 \rightarrow 14 \rightarrow 15 \rightarrow 16$ .

Fig. 2 shows the robustness of each path,  $\hat{h}_m$ , versus the critical project duration,  $T_c$ . Path 5 is far more robust than all other paths, path 1 is next in robustness, and path 2 is next. Path 3 is more robust than path 4 at critical durations less than 23.1 time units, while path 4 is more robust if the project is scheduled for more than 23.1 time units, as we see more clearly in fig. 3. Thus path 4 is the uncertainty-critical path for short values of  $T_c$ , while path 3 is uncertainty-critical for longer project durations. According to eq.(3) we see that the project robustness is determined by the robustness of path 4 for short project durations, and by path 3 for long durations.

What can we say about the **questions** raised earlier?

A reliable estimate of project duration depends on a judgment of how much robustness against task-delay is needed. Path 4 has the longest estimated duration: 21 units based on task-time estimates  $\tilde{t}$ . However, the robustness is necessarily precisely zero for this value of  $T_c$ , as we see in fig. 3. If task times are not expected to vary more than about 10% (weighted by the  $w_n$ 's) then a project duration of about 23.5 time units is reasonable. On the other hand, robustness against 30% delays requires a project duration of 31 time units.

Which paths are most vulnerable to uncertainty? Paths 3 and 4 are substantially less robust than the other paths. Between paths 3 and 4, which is more vulnerable depends on the total time allocated to the project, as explained earlier.

The allocation of resources for managing uncertainty should aim at equalizing the robustnesses of the paths. Thus, path 5 can tolerate substantial reduction in robustness without jeopardizing the timely completion of the project, while paths 3 and 4, and to a lesser extent path 2, would benefit by robustification.

Note that the strategy suggested here is to **satisfy** the project-duration requirement, eq.(2), and **maximize** the robustness to uncertainty. This is quite different from the strategy of minimizing the best-estimated project duration. The best-estimate of the project duration has zero robustness against uncertainty, and thus is not a reliable criterion for managerial decision.

**Sources.**

- Yakov Ben-Haim, 2006, *Info-Gap Decision Theory: Decisions Under Severe Uncertainty*, 2nd edition, Academic Press, London, section 3.2.6.
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- Yakov Ben-Haim and Alexander Laufer, 1998, Robust reliability of projects with activity-duration uncertainty, *ASCE Journal of Construction Engineering and Management*, 124: 125–132.