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The graph model for conflict resolution with information-gap uncertainty in preferences

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Abstract

Information-gap models, for formally modeling the uncertainty of preferences of decision makers involved in a conflict, are devised for employment with the graph model for conflict resolution. These information-gap models are designed for handling a variety of situations for expressing severe preference-uncertainty of a decision maker, including both transitive and intransitive preferences among the states or possible scenarios in a conflict. Applications of these decision technologies to the game of chicken and the Cuban Missile Crisis of 1962 illustrate how the information-gap models can be conveniently utilized in practice and how strategic insights can be gained through rigorous examination of the robustness of equilibrium solutions to uncertainty in preferences. It is also shown that uncertainty-analyses can lead to modification of a decision maker's prior preferences.

Keywords: Graph model; Conflict resolution; Info-gap uncertainty; Preference uncertainty; Cuban missile crisis

1. Introduction

Differences of opinion seem to arise whenever human beings interact with one another. Conflict is present in a commonplace situation such as a family

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trying to decide at which restaurant to have dinner and conflict is an integral part of negotiations over free trade among nations at the international level. Conflict may involve coercive action like military attacks among ethnic groups or it may involve close cooperation among organizations such as automobile manufacturers cooperating with one another when dealing with the same labor union. Whatever the case, conflict is here to stay and this reality has stimulated extensive academic research for understanding and modeling conflict throughout the rainbow of conflict situations that may take place in many different types of jurisdictions.

A key difficulty in studying any conflict situation is determining the preferences of the decision makers (DMs) involved in the dispute. A DM, for example, may try to hide his true preferences in order to attempt to obtain a better outcome for himself or a DM may attempt to portray a false impression of his preferences when behaving strategically. In some conflicts, a lack of genuine communication among DMs may result in the DMs having a hard time understanding others' objectives. Whatever the situation, there is often a high level of uncertainty about the preferences of DMs participating in a dispute. Accordingly, the main goal of this paper is to put forward a flexible procedure for incorporating uncertainty in preferences into the formal modeling of conflict. Specifically, a range of information-gap models [1,2] for preferences are constructed for use with the graph model for conflict resolution [4] in order to more realistically describe conflict situations.

Subsequent to outlining the graph model for conflict resolution in the next section, various info-gap models are developed for describing preference-uncertainties of DMs taking part in a dispute. As explained later, preferences may be expressed for each DM directly in terms of the states or possible scenarios in a conflict, or in terms of hierarchical preference statements about options or courses of action controlled by the DMs. Moreover, both transitive and intransitive preferences are considered. The subsequent section describes a decision algorithm which is a natural complement to info-gap models of uncertainty, whereby the DM satisfices while maximizing his immunity to uncertainty. Then the game of chicken is analyzed to illustrate how info-gap models are used for representing uncertainty in ordinal preferences, and to demonstrate the implications of uncertainty analysis of decision processes. Next, info-gap models based upon uncertain preference statements about options are employed with the graph model for studying the Cuban Missile Crisis. Conclusions are drawn in the final section.

2. The graph model for conflict resolution

Formal modeling techniques have been developed for systematically studying a social conflict having two or more DMs, each of whom can have multiple objectives. In particular, the graph model for conflict resolution [4] constitutes an expansion and reformulation of conflict analysis [6], which in turn is an extension of metagame analysis [7]. Other related techniques for describing human conflict include drama theory [8], which allows one to consider the role of emotions in conflict resolution, and hypergame analysis [6,9,10], which permits one to take misperceptions into account. The aforesaid approaches to strategic decision making situations can be considered as belonging to a branch of game theory that is quite distinct from more traditional methods based on the classical work of von Neumann and Morgenstern [11]. Hipel et al. [12] furnish an overview of the use of game theory models in engineering decision making while Hipel et al. [13] explain the roles of the graph model for conflict resolution and other operational research tools for refining and selecting courses of action to solve a given problem within a systems engineering context [14].

The key input information required for calibrating the graph model for a given dispute is the identities of the DMs, the possible scenarios or states that could take place, and the relative preferences of each of the DMs among the possible states. Let $N = \{1, 2, \ldots, n\}$ denote the set of DMs, where $|N| \geqslant 2$, and $U = \{u_1, u_2, \ldots, u_\mu\}$ be the set of states. A collection of finite directed graphs $\{D_i = (U, A_i), i \in N\}$ can be used to model the course of the conflict. The vertices of each graph constitute the possible states of the conflict and, hence, the vertex set U is common to all of the graphs. If DM i can unilaterally move in one step from state u_j to u_k, there is an arc with orientation from u_j to u_k in A_i. An inherent advantage of the graph model is that it can keep track of both reversible and irreversible moves. For example, if moving from one state to another involves bombing a military target, the movement would be irreversible since the damage could not be reversed after the site is bombed. The graph model can also handle common moves whereby two or more DMs can independently make unilateral moves that cause the conflict to change from one state to exactly the same other state.

For many practical applications, ordinal preference information is available for each DM such that the states are ranked from most to least preferred and there may be one or more groups of states having equally preferred states within each group. If there are no sets of equally preferred states, the preferences are said to be strictly ordinal. An assumption underlying ordinal preferences is the concept of transitivity, whereby, if a DM prefers states u_j to u_k and also u_k to u_l, this implies the DM prefers u_j to u_l. Another main advantage of the graph model is that it is theoretically designed to handle a rich range of other types of preferences including intransitivity, whereby a DM prefers state u_j to u_k and u_k to u_l, but prefers u_l to u_j. (For an indepth discussion of preference modeling the reader may wish to refer to [15] as well as in [4, Chapter 8].)

For small conflicts, a user may wish to record a conflict by drawing the directed graph for each DM, perhaps within a single integrated graph. Another way to write down a simple dispute involving only two DMs is to use a matrix whereby the row DM controls the strategies represented by each row and the column DM is in charge of the column moves. A particularly good notation for representing both simple and complex conflicts is the option form originally introduced by Howard [7] and illustrated in the two application sections in this paper. In this format, each DM controls a set of options or possible courses of actions. When the DM selects which option to implement from his or her set, a strategy is created. A specific strategy choice by every DM in the conflict forms a possible state.

Subsequent to developing a model of a given dispute, one can determine the stability of each state for each DM. A state is stable for a DM if and only if (iff) that DM has no incentive to deviate from it unilaterally, under a particular behavioral model usually referred to as a stability definition or solution concept. A state is an equilibrium or possible resolution under a particular solution concept iff all of the DMs find it stable under this stability definition. Because different DMs may behave differently under conditions of conflict, a range of solution concepts have been defined to reflect a variety of strategic decision styles. The names of solution concepts that have thus far been defined within the graph model paradigm include Nash stability, general metarationality, symmetric metarationality, sequential stability, limited-move stability and non-myopic stability. Fang et al. [4, Chapter 3] define and mathematically compare these solution concepts and provide extensive references for them. Under Nash stability, for example, a DM has little foresight whereas under non-myopic stability the DM can think about possible moves and countermoves far into the future. When a DM follows sequential stability, he is willing to take some risks and possesses medium foresight.

The theory and practice of the graph model for conflict resolution is described in the book of Fang et al. [4] as well as other research papers. To implement the graph model methodology, one can employ the decision support system called GMCRII [16,17]. This user-friendly system, which operates within a windows environment, is designed for use with transitive preferences, can be used with even very large conflicts, and calculates stability results for all of the solution concepts mentioned above.

Usually, the most difficult information to obtain in practice is the relative preference information for each DM. Although a given DM may have a good understanding of his or her own preferences, there is usually a lot of uncertainty about the preferences of others. Accordingly, a major objective of this paper is to describe how info-gap models can be employed for systematically incorporating uncertainty about preferences into a conflict study under a variety of practical situations. This new decision technology can be employed for expanding the capabilities of a conflict resolution methodology, such as the graph model for conflict resolution for finding robust and more realistic solutions to conflict problems.

The main conclusion we will reach is that uncertainty-analysis can have a profound strategic effect upon the outcome of a dispute. More specifically, a state or preference statement which is a priori highly preferred, but whose successful realization turns out to be very vulnerable to uncertainty, may be shifted to lower priority thereby changing the DM's action. Conversely, less favored expressions of preference, upon being seen as highly robust to uncertainty, may obtain higher priority ranking.

3. Info-gap models for uncertainty in preferences of decision makers

In this section, we describe two models for representing uncertainty in preference vectors, defined below. These are set-models of uncertainty, based on viewing uncertainty as an information gap: the disparity between what is known about a given preference vector and complete knowledge of the preference vector. The idea of information-gap uncertainty, as opposed to more classical ideas such as probabilistic uncertainty, is discussed elsewhere axiomatically [2], in mechanical reliability analysis [1], and in water resources management [5].

We consider two different realizations of the uncertainty model. First we only consider preference rankings which allow for rank-equivalence, but preserve transitivity of ranks. Next, we show the construction of an uncertainty model which allows both for rank-equivalence as well as non-transitivity of preferences.

3.1. Transitive ranking with possible equalities

Let J be the number of states (or preference statements, defined later). We now define a preference vector differently from that given in [6]. A preference vector π is a J-vector whose mth element is the integer preference-rank for state (or preference statement) m. That is, $\pi_m = r$ means that state m has rank r. A larger value of r implies higher preference. If states t and m have the same rank, then $\pi_t = \pi_m$ and the value of this integer is their ordinal rank. Thus, a preference vector π is a J-vector whose elements are the first $\ell(\leqslant J)$ positive integers, where ℓ will equal J only if all states have different ranks, and hence are strictly ranked.

We will iteratively define a sequence of sets of preference vectors which are successively further from the known nominal preference vector, which we denote as $\pi^{(0)}$. First, define a column J-vector e^i which has 1 in the ith position and 0 elsewhere. Thus, for any preference vector π, the vector $\pi + e^i$ raises the rank of the ith state by one. Likewise, $\pi - e^i$ lowers the rank of the ith state by one. We must add a technical proviso: these '+' and '−' operations do not raise a preference rank above J or lower it below 1. Let $\Pi^{(0)}$ be the set containing the nominal preference vector: $\Pi^{(0)} = \{\pi^{(0)}\}$. We recursively define the following sets of preference vectors:

$$\Pi^{(k)} = \left\{\pi \pm e^i \text{ for all } \pi \in \Pi^{(k-1)},\ i = 1,\ldots,J\right\}, \quad k = 1,2,\ldots \tag{1}$$

Thus $\Pi^{(k)}$ is the set of preference vectors which differ from the nominal by no more than k single preference changes.

Now we define our information-gap uncertainty model as

$$\mathscr{U}(\alpha, \pi^{(0)}) = \bigcup_{k=0}^{\alpha} \Pi^{(k)}, \quad \alpha = 0,1,2,\ldots \tag{2}$$

$\mathscr{U}(\alpha, \pi^{(0)})$ is the set of all preference vectors which differ from the known nominal preference vector by no more than α changes in rank. $\mathscr{U}(\alpha, \pi^{(0)})$ is a set-model of uncertainty. α is the information-gap uncertainty parameter. It is a non-negative integer, whose meaning is the 'distance' from the known nominal preference vector. The collection of sets $\mathscr{U}(\alpha, \pi^{(0)})$, $\alpha = 0, 1, 2, \ldots$, is a family of nested sets

$$\alpha < \beta \quad \text{implies that} \quad \mathscr{U}(\alpha, \pi^{(0)}) \subset \mathscr{U}(\beta, \pi^{(0)}). \tag{3}$$

For two preference vectors, π and π', define the *distance* between them, in terms of the number of corresponding states with different ranks, by

$$\mathrm{dis}(\pi, \pi') = \sum_{j=1}^{J} \left|\pi_j - \pi'_j\right|. \tag{4}$$

We see that $\mathscr{U}(\alpha, \pi^{(0)})$ is the set of preference vectors whose distance from the nominal preference vector $\pi^{(0)}$ is no more than α

$$\mathscr{U}(\alpha, \pi^{(0)}) = \left\{\pi\colon \mathrm{dis}(\pi, \pi^{(0)}) \leqslant \alpha\right\}, \quad \alpha = 0,1,2,\ldots \tag{5}$$

$\mathscr{U}(\alpha, \pi^{(0)})$ is defined for any non-negative integer α, indicating that it is a family of nested sets. It is this nesting, together with the distance measure, which gives the uncertainty parameter α its meaning as an information-gap.

3.2. Rank-equivalence and non-transitivity

We now describe a preference-uncertainty model which allows for rank-equivalence as well as for non-transitive preferences. We start by considering J states or preference statements as before, but now π is a $J \times J$ preference matrix. Each element of π takes one of the values -1, 0 or 1. The mth row of the preference matrix expresses the preference rank of each of the J states with respect to state m, as follows:

$$\pi_{mn} = \begin{cases} -1 & \Rightarrow \quad s_m < s_n \quad \text{(state } m \text{ is } \textit{less} \text{ preferred than state } n\text{)}, \\ 0 & \Rightarrow \quad s_m = s_n \quad \text{(state } m \text{ is } \textit{equally} \text{ preferred to state } n\text{)}, \\ 1 & \Rightarrow \quad s_m > s_n \quad \text{(state } m \text{ is } \textit{more} \text{ preferred than state } n\text{)}. \end{cases} \tag{6}$$

For instance, the matrix on the left is an example of a preference matrix π for three states, while the matrix on the right interprets the entries of π:

$$\begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} s_1 = s_1 & s_1 > s_2 & s_1 < s_3 \\ s_2 < s_1 & s_2 = s_2 & s_2 > s_3 \\ s_3 > s_1 & s_3 < s_2 & s_3 = s_3 \end{pmatrix}. \tag{7}$$

We note that all the binary relations are consistent, which means that if in row m, s_m has a particular rank relative to s_n, then in row n this same rank relation recurs. Algebraically, this means that a preference matrix will be skew-symmetric: $\pi = -\pi^{\mathrm{T}}$.

However, in Eq. (7) we note that ternary relations are not necessarily transitive. For instance, we see that

$$\begin{matrix} s_1 > s_2 & \text{and} & s_2 > s_3 & \text{but} & s_3 > s_1 \\ (\pi_{12} = 1) & & (\pi_{23} = 1) & & (\pi_{31} = 1) \end{matrix} \tag{8}$$

However, since binary relations are consistent, ternary *non*-transitivity is also consistent. For instance

$$\begin{matrix} s_1 < s_3 & \text{and} & s_3 < s_2 & \text{but} & s_2 < s_1 \\ (\pi_{13} = -1) & & (\pi_{32} = -1) & & (\pi_{21} = -1) \end{matrix} \tag{9}$$

which is consistent with the non-transitivity of the relations in Eq. (8).

Now we construct a nested-set uncertainty model based on the information-gap concept. We first define a non-commutative operator which performs single preference changes on preference matrices. When the operator $\mathcal{O}_{mn}$, where $m \neq n$, acts on a preference matrix π, it produces a matrix with the same dimensions whose elements are defined as

$$[\mathcal{O}_{mn} \circ \pi]_{ij} = \begin{cases} \pi_{ij} \\ \quad \text{if } \ (i,j) \neq (m,n) \text{ and if } (i,j) \neq (n,m), \\ -\pi_{ij} + (1 - |\pi_{ij}|)\,\frac{i-j}{m-n} \\ \quad \text{if } \ (i,j) = (m,n) \text{ or if } (i,j) = (n,m). \end{cases} \tag{10}$$

$\mathcal{O}_{mn}$ acts only on the (m,n) and (n,m) elements of π, leaving all other elements unchanged. If π_{mn} and π_{nm} are non-zero then their signs are changed; if they are zero then $\pi_{mn} = +1$ and $\pi_{nm} = -1$. Note that $\mathcal{O}_{mn}$ is defined only for $m \neq n$.

For example, consider the preference matrix

$$\pi = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}. \tag{11}$$

When $\mathcal{O}_{1,2}$ or $\mathcal{O}_{2,1}$ acts on π the $(1,2)$ and $(2,1)$ elements are switched and all other elements are preserved

$$\mathcal{O}_{1,2} \circ \pi = \mathcal{O}_{2,1} \circ \pi = \begin{pmatrix} 0 & -\mathbf{1} & 0 \\ +\mathbf{1} & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}. \tag{12}$$

Likewise, $\mathcal{O}_{2,3}$ or $\mathcal{O}_{3,2}$ modifies only the $(2,3)$ and $(3,2)$ elements

$$\mathcal{O}_{2,3} \circ \pi = \mathcal{O}_{3,2} \circ \pi = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -\mathbf{1} \\ 0 & +\mathbf{1} & 0 \end{pmatrix}, \tag{13}$$

$\mathcal{O}_{1,3}$ modifies two of the zero elements

$$\mathcal{O}_{1,3} \circ \pi = \begin{pmatrix} 0 & 1 & +\mathbf{1} \\ -1 & 0 & 1 \\ -\mathbf{1} & -1 & 0 \end{pmatrix}. \tag{14}$$

Finally, $\mathcal{O}_{3,1}$ modifies the same two zero elements, but in the opposite sense

$$\mathcal{O}_{3,1} \circ \pi = \begin{pmatrix} 0 & 1 & -\mathbf{1} \\ -1 & 0 & 1 \\ +\mathbf{1} & -1 & 0 \end{pmatrix}. \tag{15}$$

We now use the $\mathcal{O}_{mn}$ operator to define the info-gap model for uncertainty in the preference matrix. As before, define $\Pi^{(0)} = \{\pi^{(0)}\}$, where $\pi^{(0)}$ is the nominal preference matrix. We recursively define the following sets of preference matrices:

$$\begin{aligned} &\Pi^{(k)} = \left\{\mathcal{O}_{mn} \circ \pi \text{ for all } \pi \in \Pi^{(k-1)},\ m,n = 1,\ldots,J,\ m \neq n, \right\}, \\ &k = 1,2,\ldots \end{aligned} \tag{16}$$

$\Pi^{(k)}$ is the set of preference matrices which differ from the preference matrices in $\Pi^{(k-1)}$ by no more than one change in preference. The uncertainty model is defined as in Eq. (2), so that $\mathcal{U}(\alpha, \pi^{(0)})$ is a family of nested sets, for $\alpha = 0,1,2,\ldots$ The uncertainty parameter α corresponds to the distance from the nominal preference matrix in terms of the number of preference changes.

4. Robust rationality with uncertain preferences

We now define several concepts of robustness. We consider N DMs, where $\mathscr{U}_m(\alpha, \pi_m^{(0)})$ is the uncertainty model for the preferences of the mth DM. $\mathscr{U}_m(\alpha, \pi_m^{(0)})$ is either of the two classes of uncertainty models described earlier. The uncertainty models of the various DMs do not have to be of the same class. We consider three types of robustness measures: robustness of possible resolutions of the conflict as a whole, robustness of a strategy of an individual DM, and robustness of a strategy with a specified minimum acceptable ordinal utility.

4.1. Robustness of the conflict

It will usually be true that at least one choice of options by the N DMs constitutes an equilibrium resolution of the conflict, based on the nominal preferences of the DMs. In fact, when using the solution concept of sequential stability, at least one equilibrium always exists for ordinal games [4,6]. In practice, there may be more than one equilibrium solution, and the set of all equilibrium solutions, based on the nominal preferences, is called the nominal equilibrium solution set and is denoted $\mathscr{Z}(\pi_1^{(0)}, \ldots, \pi_N^{(0)})$ where $\pi_n^{(0)}$ represents the nominal preferences of the nth DM. For arbitrary preferences of the DMs, $\pi_1, \ldots, \pi_N$, the set of equilibrium solutions is denoted as $\mathscr{Z}(\pi_1, \ldots, \pi_N)$.

The *overall robustness*, $\widehat{\alpha}$, is the greatest value of the uncertainty parameter for which the set of equilibrium solutions is the same as the nominal equilibrium set. More precisely

$$\widehat{\alpha} = \max \Big\{ \alpha\colon \mathscr{Z}(\pi_1, \ldots, \pi_N) = \mathscr{Z}(\pi_1^{(0)}, \ldots, \pi_N^{(0)}) \\ \text{for all } \pi_m \in \mathscr{U}_m(\alpha, \pi_m^{(0)}),\ m = 1, \ldots, N \Big\}. \tag{17}$$

When $\widehat{\alpha}$ is large the equilibrium solutions are robust to preference-uncertainty. On the other hand, if $\widehat{\alpha}$ is small then equilibrium resolution of the conflict is quite vulnerable to the incomplete information available to the DMs.

A *conditional robustness* is the greatest value of the uncertainty parameter for which the set of equilibrium solutions is the same as the nominal equilibrium set, for fixed values of the preferences of some of the DMs. Let $\mathscr{I}$ be a set of indices of some but not all of the DMs. The robustness conditioned on fixed preferences for DMs indexed in $\mathscr{I}$ is

$$\widehat{\alpha}(\pi_m, m \in \mathscr{I}) = \max \Big\{ \alpha\colon \mathscr{Z}(\pi_1, \ldots, \pi_N) = \mathscr{Z}\left(\pi_1^{(0)}, \ldots, \pi_N^{(0)}\right) \\ \text{for all } \pi_n \in \mathscr{U}_n(\alpha, \pi_n^{(0)}),\ n \notin \mathscr{I} \Big\} \tag{18}$$

We see that the overall robustness is simply the conditional robustness when the index set $\mathscr{I}$ is empty.

4.2. Robustness of a strategy

We consider a situation in which each DM may use a *strategy* by which he influences his own as well as his opponents' reachable states. In a game-theoretic setting, the strategy at each stage of the game is simply a choice from the set of options available to this DM. We will define two robustnesses associated with the choice of a strategy.

The equilibrium solution set $\mathscr{X}(\pi_1, \ldots, \pi_N)$ is defined as before. Let $\mathscr{X}_m(\pi_1, \ldots, \pi_N \mid \sigma)$ be the subset of $\mathscr{X}(\pi_1, \ldots, \pi_N)$ whose elements all entail the implementation of strategy σ by DM m.

The *overall robustness* of strategy σ is the greatest value of the uncertainty parameter for which the equilibrium solution subset $\mathscr{X}_m(\pi_1, \ldots, \pi_N \mid \sigma)$ is the same as the nominal equilibrium solution subset, if m implements σ

$$\gamma_m(\sigma) = \max\left\{\alpha\colon \mathscr{X}_m(\pi_1, \ldots, \pi_N \mid \sigma) = \mathscr{X}_m\left(\pi_1^{(0)}, \ldots, \pi_N^{(0)} \mid \sigma\right)\right. \\ \left. \text{for all } \pi_m \in \mathscr{U}_m(\alpha, \pi_m^{(0)}),\ m = 1, \ldots, N\right\}. \quad (19)$$

We see that $\gamma_m(\sigma)$ is related to the overall robustness, where the additional constraint is imposed that DM m implements strategy σ.

A quantity similar to the conditional robustness $\widehat{\alpha}(\pi_m, m \in \mathscr{I})$ is defined by imposing the further condition that m implements σ

$$\gamma_m(\pi_m, m \in \mathscr{I} \mid \sigma) = \max\left\{\alpha\colon \mathscr{X}_m(\pi_1, \ldots, \pi_N \mid \sigma) = \mathscr{X}\left(\pi_1^{(0)}, \ldots, \pi_N^{(0)} \mid \sigma\right)\right. \\ \left. \text{for all } \pi_n \in \mathscr{U}_n(\alpha, \pi_n^{(0)}),\ n \notin \mathscr{I}\right\}. \quad (20)$$

The immediate use of the robustness $\gamma_m(\pi_m, m \in \mathscr{I} \mid \sigma)$ is in evaluating and choosing between alternative strategies. Suppose that DM m is contemplating two alternative strategies, σ_1 and σ_2. Suppose that $\gamma_m(\pi_m, m \in \mathscr{I} \mid \sigma_1) \gg \gamma_m(\pi_m, m \in \mathscr{I} \mid \sigma_2)$, meaning that the first strategy is much more robust to uncertainty than the second strategy. Consequently, the DM will be tempted to adopt σ_1 rather than σ_2. This conclusion is reasonable even if the nominal preference of DM m for σ_1 is less than his nominal preference for σ_2. In other words, the analysis of the robustness to uncertainty can alter a DM's preferences. We will see an illustration of this in the subsequent examples.

4.3. Robustness of a strategy with minimum acceptable utility

A DM may recognize that he is unable to obtain the state which he most prefers as the outcome of the conflict, or that attainment of this state may be very vulnerable to uncertainty. In terms of ordinal preferences, he may be unable to achieve his preference of highest rank. In light of this realization, he may wish to identify a *critical utility,* u_{cr} which is an integer corresponding to the least ordinal preference he is willing to accept. If he insists on the optimal, then $u_{cr} = J$; if he is more modest, he will choose a value less than J for u_{cr}.

When DM m contemplates a strategy σ, he may wish to know how robust the implementation of this strategy is, to uncertainty in the preferences of the other DMs. Also, he may wish to know how the robustness of a strategy varies with u_{cr}, his least-acceptable ordinal utility.

Let us define $u_m(\pi_1, \ldots, \pi_N \mid \sigma)$ as the least ordinal utility which DM could achieve, at equilibrium, if he implements strategy σ and if the preferences of the DMs are $\pi_1, \ldots, \pi_N$. The robustness of strategy σ for DM m, when he demands ordinal utility no less than u_{cr}, is the greatest value of the uncertainty parameter which guarantees utility no less than u_{cr}

$$\zeta_m(u_{cr}, \sigma) = \left\{ \alpha\colon u_m(\pi_1, \ldots, \pi_N \mid \sigma) \geqslant u_{cr} \text{ for all } \pi \in \mathscr{U}_m\left(\alpha, \pi_m^{(0)}\right), \right. \\ \left. m = 1, \ldots, N \right\}. \tag{21}$$

The immediate use of the robustness $\zeta_m(u_{cr}, \sigma)$ is in evaluating and choosing between alternative strategies, similar to the use of $\gamma_m(\pi_m, m \in \mathscr{I} \mid \sigma)$. In other words, $\zeta_m(u_{cr}, \sigma)$ can be used to evaluate and revise a DM's preferences in light of the uncertainty of his knowledge.

Another important property of this robustness is the trade-off between demanded utility and robustness. One would expect that when u_{cr} is very large, meaning that the DM is demanding a very favorable outcome, then the robustness of any strategy σ will be low. That is, $\zeta_m(u_{cr}, \sigma)$ should decrease monotonically with u_{cr} as implied by the 'gambler's theorem' in [3].

5. Example: the game of chicken

In this section we will perform an uncertainty analysis of the game of 'chicken'. We will use the robustness defined in Eq. (20) to demonstrate that the analysis of uncertainty can alter a DM's priorities, and can lead him to decisions which are quite different from those made in the absence of uncertainty.

The game of chicken has been extensively studied as a prototype of other adversary proceedings [18]. In chicken, two car drivers race towards each other at high speed. Each driver has the choice of either swerving to avoid collision, or continuing to drive straight ahead. Table 1 displays the game of chicken

Table 1
Four states in the game of chicken

	State 1	State 2	State 3	State 4
Driver 1				
1. Swerve	N	N	Y	Y
Driver 2				
2. Swerve	N	Y	N	Y

written in option form. The left-hand side of the table lists each of the two DMs in chicken, followed by the single option, or binary decision, called 'swerve', which each DM controls. The four columns of Y's and N's represent the four possible states that could occur. A 'Y' indicates 'yes' the option is selected by the DM controlling it, while 'N' means the option is not taken. Consider, for example, state 3 in Table 1: the first driver swerves while the second driver does not and thereby wins the game.

When the second driver remains fixed on choosing N for his strategy, the first driver can unilaterally cause the game to move from state 1 to 3 by changing his selected option from N to Y. If driver 1 is allowed to go back onto the road after swerving, he also controls the movement from state 3 to 1. As can be seen, driver 1 also is in charge of the movement between states 2 and 4 when the second driver has a fixed strategy of Y. Following similar arguments, driver 2 controls the unilateral movements between states 1 and 2 as well as states 3 and 4. Fig. 1 portrays the directed graph of unilateral moves in one step for each of the DMs.

Let us play the part of driver 1, and adopt the preference vector $\pi_1 = (1, 4, 2, 3)$, indicating that our greatest preference is for state 2, in which our opponent swerves but we do not. Our next preference is for state 4, then for state 3 and our least preferred outcome is state 1. This is a reasonable and rather humane preference vector. Our first preference is for state 2, in which only our opponent swerves and is disgraced. However, by giving our next preference to state 4, we are emphasizing an 'amicable' outcome, in which both drivers swerve, both survive, and both are 'disgraced' to the same degree.

Now suppose that our opponent's preference vector is $\pi_2 = (1, 1, 4, 1)$, which is quite different from ours. His only preference is for disgracing us by forcing us to swerve; he is indifferent to any other outcome.

As noted earlier, there are many definitions of stability and equilibrium of games [4, Chapter 3]. We consider the following version of Nash stability. A state is *stable* for a given DM if he has no incentive for moving from that state. That is, state m is Nash stable for DM i if i's preference for state m is no less

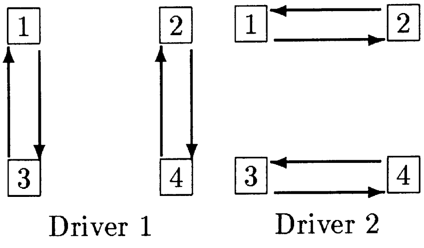

Fig. 1. Unilateral moves by the two drivers in the game of chicken.

than his preference for any other accessible state for fixed strategy choices of the other DMs. State m is stable for DM i if

$$\pi_{m,i} \geqslant \pi_{k,i} \tag{22}$$

for all states k in which the strategy choices of the other DMs are the same as in state m.

A state is an *equilibrium* if it is stable for both DMs. Table 2 shows the stable and equilibrium states for the game of chicken, with the preference vectors we have adopted. We see that two states, 2 and 3, are both equilibrium outcomes, since neither DM has an incentive to move from either of these states, according to the specified preference vectors and Nash stability definition.

State 3, for example, is Nash stable for driver 1 because he prefers state 3 more than state 1 and from 3 he can only move to 1. Notice for both states 3 and 1 driver 2 has a fixed strategy of N for not swerving. From driver 2's point of view, state 3 is Nash stable for him because he prefers state 3 over state 4, where driver 1 has a fixed strategy of Y. Since state 3 is Nash stable for both drivers, it constitutes a Nash equilibrium.

Now we consider the analysis of uncertainty, employing the robustness of Eq. (20) and adopting the role of driver 1. We know our own preference vector precisely, $\pi_1 = (1,4,2,3)$. However, our knowledge of our opponent's preferences is uncertain, and our nominal preference vector for our opponent is $\pi_2^{(0)} = (1,1,4,1)$.

From Table 2 we see that the set of equilibrium states, based on the nominal preference vectors, contains states 2 and 3 only

$$\mathscr{X}(\pi_1, \pi_2^{(0)}) = \{2,3\}. \tag{23}$$

So, the set of equilibrium solutions constrained to entail option 'No' by driver 1 is the subset of $\mathscr{X}(\pi_1, \pi_2^{(0)})$ which contains state 2

$$\mathscr{X}_1(\pi_1, \pi_2^{(0)} \mid \sigma = \text{No}) = \{2\}. \tag{24}$$

Likewise, the set of equilibrium solutions constrained to entail option 'Yes' by driver 1 is the subset of $\mathscr{X}(\pi_1, \pi_2^{(0)})$ which contains state 3

$$\mathscr{X}_1(\pi_1, \pi_2^{(0)} \mid \sigma = \text{Yes}) = \{3\}. \tag{25}$$

Table 2
Nash stability for the game of chicken, with the preference vectors $\pi_1 = (1,4,2,3)$ and $\pi_2 = (1,1,4,1)$

State	DM1	DM2	Outcome
1	Unstable	Stable	Not equilibrium
2	Stable	Stable	Equilibrium
3	Stable	Stable	Equilibrium
4	Unstable	Unstable	Not equilibrium

In a similar fashion, we can construct the collection of all equilibrium sets, for all the preference vectors of driver 2, as the level of uncertainty increases. This is shown in Table 3. When $\alpha = 0$, implying no uncertainty in the preference vector for driver 2, the set of equilibrium solutions contains states 2 and 3, as we expect from Table 2. The constrained equilibrium sets contain states 2 or 3, for the two different options considered by driver 1, as explained in connection with Eqs. (24) and (25).

When $\alpha = 1$, implying uncertainty of no more than one step away from $\pi_2^{(0)}$, more than a single equilibrium solution set exists: the previous set as well as the set containing only state 3. Now the constrained solution set for option $\sigma =$ 'Yes' contains only state 3. However, for the option $\sigma =$ 'No', one constrained equilibrium set is $\{2\}$ while the other constrained set is empty, implying that the desired option does not lead to equilibrium with some preference vector for driver 2 at uncertainty $\alpha = 1$.

This situation recurs also at uncertainty levels 2 and 3. Finally, at uncertainty level $\alpha = 4$ we find that one of the constrained equilibrium solution sets for option $\sigma =$'Yes' is empty. This means that a preference vector for driver 2 at uncertainty $\alpha = 4$ exists which does not allow equilibrium if driver 1 chooses this option.

What we learn from Table 3 is that if driver 1 chooses option $\sigma =$'Yes', then he will always reach an equilibrium, for any preference vector for driver 2 up to and including uncertainty level $\alpha = 3$. On the other hand, one cannot guarantee that the option $\sigma =$'No' will be an equilibrium point at uncertainty level $\alpha = 1$. In short, the conclusion from Table 3 is that the conditional robustness, Eq. (20), for the two options available to driver 1 is:

$$\gamma_1(\pi_1 \mid \sigma = \text{Yes}) = 3, \tag{26}$$

$$\gamma_1(\pi_1 \mid \sigma = \text{No}) = 0. \tag{27}$$

In other words, option 'Yes' is much more robust than option 'No', with respect to driver 1's uncertainty about his opponent's preferences. On the one hand, state 2 (entailing option 'No') is preferred by driver 1 over state 3

Table 3
Equilibrium sets with increasing uncertainty with preference vectors $\pi_1 = (1, 4, 2, 3)$ and $\pi_2^{(0)} = (1, 1, 4, 1)$

Level of uncertainty (α)	Equilibrium sets $\mathscr{X}(\pi_1, \pi_2)$, $\pi_2 \in \mathscr{U}(\alpha, \pi_2^{(0)})$	Constrained equilibrium sets $\mathscr{X}_1(\pi_1, \pi_2 \mid \sigma = \text{Yes})$, $\pi_2 \in \mathscr{U}(\alpha, \pi_2^{(0)})$	Constrained equilibrium sets $\mathscr{X}_1(\pi_1, \pi_2 \mid \sigma = \text{No})$, $\pi_2 \in \mathscr{U}(\alpha, \pi_2^{(0)})$
0	$\{2,3\}$	$\{3\}$	$\{2\}$
1	$\{2,3\}$, $\{3\}$	$\{3\}$	$\{2\}$, $\emptyset$
2	$\{2,3\}$, $\{3\}$	$\{3\}$	$\{2\}$, $\emptyset$
3	$\{2,3\}$, $\{3\}$	$\{3\}$	$\{2\}$, $\emptyset$
4	$\{2,3\}$, $\{2\}$, $\{3\}$	$\{3\}$, $\emptyset$	$\{2\}$, $\emptyset$

(entailing option 'Yes'). However, the analysis of uncertainty has shown that the achievement of equilibrium for state 2 is much more fragile to uncertainty than achievement of equilibrium for state 3. Consequently, driver 1 may well select option 'Yes' (state 3) rather than 'No' (state 2). What we see, in short, is that the analysis of uncertainty can fundamentally alter a DM's preferences.

6. Example: Cuban Missile Crisis

6.1. Background

In this application, a version of the info-gap model in Eq. (5) is employed to demonstrate how changes in preferences for the Americans can have a dramatic effect upon the outcome of the Cuban Missile Crisis that erupted between the United States (US) and the Union of the Soviet Socialist Republics (USSR) in October 1962. The events leading up to this crisis started with the overthrow of the Batista regime in Cuba by Fidel Castro in 1959. The US was angered by the subsequent confiscation of American property in Cuba and the perception of a communist military threat so close to home. This culminated in the ill-advised American-sponsored Bay of Pigs invasion in April 1961 in which Cuban exiles failed to gain a foothold in Cuba. After the Bay of Pigs fiasco, American President John F. Kennedy publicly committed his administration never to tolerate offensive missiles in Cuba.

On 14 October 1962, American aerial reconnaissance discovered irrefutable evidence of Soviet offensive missiles being installed at various sites in Cuba. In order to obtain wise advice on what to do from as many reliable sources as possible, President Kennedy created the Executive Committee of the National Security Council. This committee included major cabinet and government agency officers with principal responsibilities for political and military decisions, representatives of key segments of the public, and some special advisors. The Executive Committee put forward a number of possible actions in response to the Soviet threat including performing no aggressive action, carrying out surgical air strikes against the missile bases in Cuba, and imposing a naval blockade of Cuba by turning back ships carrying military supplies to Cuba [19,20].

Premier Nikita Kruschev of the USSR had to decide whether or not to withdraw Soviet missiles from Cuba. He could also escalate the conflict through coercive actions such as putting pressure on West Berlin, attacking US naval vessels, bombing Southeastern American targets from Cuba or initiating an ICBM (Intercontinental Ballistic Missile) assault on the US. Because of the restraint and wisdom exercised by the heads of both superpowers, the Cuban Missile crisis did not result in nuclear war. Instead, the US adopted a strategy of blockading military shipments to Cuba, and the USSR withdrew the offensive missiles [19,20]. Up to the present time, the Americans have kept their promise not to carry out a military invasion of Cuba.

6.2. Decision makers, options and feasible states

The left-hand side of Table 4 lists each of the two major DMs in the Cuban conflict as well as the options under the control of each stakeholder. As can be seen, the US controls the option of executing a surgical air strike (written as Air Strike in Table 4) as well as implementing a naval blockade of Cuba to prevent further missiles to be shipped to Cuba by the USSR (Blockade). The USSR has the power to withdraw its missiles from Cuba (Withdraw) or escalate the conflict (Escalate). Cuba is not included as a DM in this model since it possessed no real power to exercise over the USSR or the US. The DMs and options shown in Table 4 are the same as those put forward by Fraser and Hipel [6] who analyzed this controversy using conflict analysis. In this paper, the decision support system GMCRII [16,17] was used to carry out all modeling and analyses according to the methodology of the graph model for conflict resolution [4,23].

The columns of Y's and N's in Table 4 constitute the 12 feasible states for this model of the Cuban Missile Crisis. Notice that there are no states for which the USSR withdraws and escalates at the same time since these were deemed to be infeasible. State 7 is the historical equilibrium or resolution to the conflict. At state 7, the US has followed the strategy of not performing an air strike and selecting the option of blockading Cuba. The USSR has chosen the strategy of withdrawing its missiles and not escalating the conflict.

For any feasible state, a particular DM may be able to unilaterally cause a transition from one state to another state by changing his or her option selection. For example, the US controls the unilateral move from state 1 to 3 by choosing the option of blockade. Since this move is unilateral on the part of the US, the USSR has the same strategy of doing nothing in both states 1 and 3. GMCRII automatically calculates all possible unilateral moves for a DM in a conflict while it prompts the user to indicate any irreversible moves. For

Table 4
Four options and 12 feasible states in the model of the Cuban Missile Crisis

DMs and options	States											
	1	2	3	4	5	6	7	8	9	10	11	12
US												
1. Air Strike	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y
2. Blockade	N	N	Y	Y	N	N	Y	Y	N	N	Y	Y
USSR												
3. Withdraw	N	N	N	N	Y	Y	Y	Y	N	N	N	N
4. Escalate	N	N	N	N	N	N	N	N	Y	Y	Y	Y

instance, one may wish to do an analysis in which the US can go from no air strike to bombing but not the reverse.

6.3. Preferences

Usually the most difficult hurdle to overcome when calibrating a decision model is obtaining accurate preference information. Even though the graph model for conflict resolution only requires at most ordinal preference information for each DM, there is still often a high level of uncertainty about preferences. For the case of the Cuban Missile Crisis, for example, the USSR was uncertain about the actual preferences of the US and it did not expect the US to react as strongly as it did. Therefore, later in this section, an info-gap model is developed for representing the uncertainty in the ordinal preferences of the US as perceived by the USSR.

Because pairwise comparisons of feasible states can be very time consuming for ranking states for a given DM, even for smaller disputes, a more natural and easier procedure is required. The Option Prioritizing approach used in GMCRII constitutes a generalization of the 'preference tree' technique originally devised by Fraser and Hipel [21] and later expanded by Peng et al. [22]. The left column in Table 5 lists how preferences are expressed hierarchically for the US in terms of preference statements about options, which are then used in an algorithm to rank the states. (Recall the distinction between options, of which there are 4, and feasible states which are 12 in number.) For instance, the statement '−2 if 3' means 'Option 2 is not implemented if option 3 is implemented'. Of the 9 preference statements in the left column of Table 5, this statement is 4th in ordinal rank.

The statement '3', meaning that option 3 is implemented, is the most preferred, which means that the US most prefers that the USSR simply withdraw its missiles from Cuba by selecting option 3. The next preferred preference

Table 5
Two cases of option prioritizing used in the info-gap model

Original US preference statements	Switching rows 3 and 4
3	3
−4	−4
−1 if 3	−2 if 3
−2 if 3	−1 if 3
1 if 4	1 if 4
2 if 4	2 if 4
1 or 2 if −3 & −4	1 or 2 if −3 & −4
−1 if −3 & −4	−1 if −3 & −4
−2 if −3 & −4	−2 if −3 & −4
States 5 and 7 are sequentially stable equilibria	State 5 is a sequentially stable equilibrium

statement is '−4' meaning that the US prefers that option 4 not be taken. The preference statement with rank 3 is '−1 if 3' meaning that the US prefers that option 1 be rejected if option 3 is taken. The 7th-ranked preference statement is '1 or 2 if −3 & −4' which indicates that the US prefers carrying out an air strike (1) or blockade (2) if the USSR does not withdraw (−3) and does not escalate (−4).

The ordinal ranking of preference statements, as in the left column of Table 5, is used to deduce the ordinal ranking of states (with possible rank equalities). This is done automatically by GMCRII using the following algorithm. The results are shown in Table 6.

Each preference statement takes a truth value, either True (T) or False (F), for a particular state. For example, the preference statement '−4' is true (realized) if state 8 occurs, since option 4 is 'N' in state 8 as indicated in Table 4.

Each state, u, is ranked according to a score, $\Psi(u)$, assigned to state u according to the truth values of the preference statements. Let k be the total number of preference statements for the DM under consideration. (In Table 5 $k = 9$.) Denote the preference statements by Ω_j, $0 \leqslant j \leqslant k$. If statement Ω_j is true when state u occurs we write: $\Omega_j(u) = T$. Now define the functions

$$\Psi_j(u) = \begin{cases} 2^{k-j} & \text{if } \Omega_j(u) = T, \\ 0 & \text{otherwise} \end{cases} \tag{28}$$

and

$$\Psi(u) = \sum_{j=1}^{k} \Psi_j(u). \tag{29}$$

The states u are then sorted according to their scores $\Psi(u)$ in order to obtain the ordinal ranking of states for the DM [22]. Thus, in Table 6 we see that state 5 is most preferred and state 9 is least preferred by the US, based on the preference-statement ranks given in the left column of Table 5.

For the case of the USSR, the hierarchical list of preference statements is −4, 1 if 4, 2 if 4, −1 if −4, −2 if −4, 3 iff 1 or 2. The option prioritizing algorithm in GMCRII then ranks the states from most to least preferred using the state numbers as (1, 5, 7, 3, 6, 2, 8, 4, 12, 10, 11, 9).

Table 6
Ranking of states from most preferred on the left to least preferred on the right using option prioritizing for the US

DMs and options	5	7	6	8	3	2	4	1	12	10	11	9
US												
1. Air Strike	N	N	Y	Y	N	Y	Y	N	Y	Y	N	N
2. Blockade	N	Y	N	Y	Y	N	Y	N	Y	N	Y	N
USSR												
3. Withdraw	Y	Y	Y	Y	N	N	N	N	N	N	N	N
4. Escalate	N	N	N	N	N	N	N	N	Y	Y	Y	Y

6.4. Stability analysis and results

As pointed out in the earlier section on the graph model for conflict resolution, each state in a conflict can be analyzed for stability for each DM according to a range of mathematically defined solution concepts of possible human behavior under conditions of conflict. A particularly useful solution concept is sequential stability [4,6] which possesses medium foresight in terms of moves and counter-moves by opponents against a possible unilateral improvement by the DM under consideration. Sequential stability reflects a DM who is willing to take some strategic risk and seeks satisficing solutions [16]. Moreover, when the preferences of the DMs are ordinal, an existence theorem states that sequential stability always produces at least one equilibrium. Therefore, the stability results and interpretation for the Cuban Missile Crisis are explained here with respect to sequential stability.

Definition of sequential stability (for the case of two DMs). For a DM $i \in N$, where $N = \{i, j\}$, a state $k \in U$ is sequentially stable for DM i iff for every $k_1 \in S_i^*(k)$ there exists $k_2 \in S_j^*(k_1)$ with $P_i(k_2) \leqslant P_i(k)$, where $N = \{i, j\}$ is the set of DMs, U is the set of states, $S_i^*(k)$ is the set of unilateral improvements (UIs) for DM i from state k, $S_j^*(k_1)$ is the set of UI's for DM j from state k_1, and $P_i(k)$ is the payoff or relative preference of state k for DM i. A rational or Nash stable state is actually a subset of the sequential stability definition for the special situation for which the set $S_i^*(k)$ is empty. As is the situation for the theoretical definitions of all the solution concepts mentioned earlier, special algorithms are programmed within the engine of the decision support system GMCRII to calculate sequential stability for a particular state and given DM.

For the Cuban Missile Crisis, the three states that are sequentially stable for both the US and USSR and, hence, form sequentially stable equilibria, are states 5, 6 and 7. As can be seen in Table 4, at state 5, the USSR withdraws its missiles from Cuba on its own. At the other two states, the USSR also withdraws its missiles but the US carries out an air strike at state 6 and imposes a naval blockade at state 7. In fact, state 7 is what happened historically. The conflict started at the status quo state 1 at which neither the US nor USSR has selected any of its options. By unilaterally imposing a blockade, the US caused the conflict to move from state 1 to 3. Finally, by agreeing to withdraw its missiles, the USSR moved the conflict from state 3 to state 7 which is the historical equilibrium and is sequentially stable.

6.5. Info-gap model for uncertainty in preferences

There is historical evidence to suggest that the USSR did not correctly understand the true preferences of the US. In particular, because the US had performed so poorly at the Bay of Pigs invasion, as well as other reasons, Premier Kruschev expected a weak response from the US to the placement of

Soviet missiles in Cuba [19,20]. One approach for modeling a situation like this is to employ hypergame analysis in which one or more DMs has misperceptions about the true nature of the conflict [9,10], as has already been done for the Cuban Missile Crisis [6,23].

An alternative approach is to use an info-gap model to represent the high uncertainty in the USSR perception of the US preferences. Since state 6 involves coercive military bombing by the US, the two sequentially stable equilibria that are viewed as feasible states by the USSR are states 5 and 7. Therefore, one would like to determine the robustness of these two equilibria to the USSR's uncertainty about the US preferences.

In Eq. (18), let the set of nominal equilibria of interest be

$$\mathscr{X}\left(\pi_1^{(0)}, \pi_2^{(0)}\right) = \{5, 7\}, \tag{30}$$

where $\pi_1^{(0)}$ is the list of hierarchical preference statements given for the US in the left column of Table 5 and $\pi_2^{(0)}$ is the preference statement for the USSR mentioned in the text. Here, the robustness is conditioned upon the preference statements of the USSR remaining fixed. So, referring to Eq. (18), we are calculating $\widehat{\alpha}(\pi_2^{(0)})$. To construct $\mathscr{U}_1(\alpha, \pi_1^{(0)})$ according to Eq. (5), for $\alpha = 1$, one must perform all single-step preference modifications on the nominal US preference vector in the left column of Table 5. By requiring strict ranking of preference statements, there are eight possible ways in which the preference list can be changed by switching two adjoining preference statements one at a time. For example, the first single switch to make in Table 5 would be to put the −4 above the 3 to form the list of preference statements from which the ranking of states could be obtained for the US. A stability analysis could then be carried out with these new US preferences and the fixed Russian preferences to see if states 5 and 7 remain as sequentially stable equilibria.

For all eight possible exchanges of adjoining preference statements for the US list of preference statements, one case caused state 7 to no longer be an equilibrium point according to sequential stability. This situation is given in the right column of Table 5. For the case on the right in Table 5, rows 3 and 4 are reversed when compared to the left column. As noted at the bottom of the right column in Table 5, only state 5 is an equilibrium for the preference statements given above. However, to reach state 5 the USSR must unilaterally disimprove from state 1, which may be difficult to do without some enticement or side-payment by the US. Notice that rows 3 and 4 in the preference statements in the right column of Table 5 state that the US 'prefers not to blockade if the Russians withdraw' more than 'not to have air strikes if the Russians withdraw'. This change in preference causes state 7 no longer to be a sequentially stable equilibrium.

In other words, the nominal equilibrium set is 'lost' at the first level of uncertainty, $\alpha = 1$. The robustness in Eq. (18) is zero

$$\widehat{\alpha}\left(\pi_2^{(0)}\right) = 0. \tag{31}$$

This means that the USSR's preference vector $\pi_2^{(0)}$ entails great vulnerability, of the equilibrium solution set, to uncertainty in the perceptions by the USSR of the US preferences.

In summary, the results of the info-gap modeling show that equilibrium state 7 is not robust to changes in the US preferences. Therefore, the USSR would be advised in this case to be sure that it has an accurate perception of American intentions. In fact, if the USSR's perceptions of the American preferences were the same as the right column in Table 5, it would not foresee state 7 as a possible equilibrium. The USSR would also not be motivated to unilaterally disimprove from state 1 to 5, without suitable enticement from the US.

7. Conclusions

A variety of info-gap models of uncertainty are proposed for combining with the graph model for conflict resolution in order to determine the strategic consequences of uncertainty that may be present in the preferences of one or more DMs involved in a dispute. As illustrated by the game of chicken and the Cuban Missile Crisis, uncertainty in preferences can have dramatic effects upon the possible resolutions of conflict situations and can thereby influence the behavior of participants. Because the info-gap model possesses a rigorous axiomatic basis and can be readily applied in practice, it furnishes a systematic procedure for investigating concepts of robustness under uncertainty of preferences. Although, from one point of view, info-gap modeling could be interpreted as being a comprehensive approach to executing sensitivity analyses, it does in fact go beyond traditional sensitivity analysis and constitutes a mathematical procedure for formally modeling and analyzing large strategic uncertainty. Besides uncertainty of preferences, info-gap models could be designed for tackling other kinds of uncertainties arising under conditions of conflict such as option and strategy selections as well as coalition formation [24].

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