Info-Gap Robust-Satisficing Monetary Policy: Modelling and Managing Ignorance

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Contents

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Abstract

Given a macroeconomic model, and a class of policy rules, is there a choice of policy which can be depended upon to achieve specified goals, in light of unknown model errors? Info-gap decision theory is used to develop a response to this question. We describe the choice of Taylor coefficients in a low-order model of the U.S. economy whose coefficients are highly uncertain. An info-gap model non-probabilistically quantifies this Knightian uncertainty. We show that choosing the Taylor coefficients to maximize the economic performance necessarily minimizes the robustness to model (and other) uncertainty. Therefore the performance (e.g. price stability or output gap) should be *satisficed* rather than optimized. What is *optimized* here is the robustness to uncertainty in the economic model.

1 Introduction

This paper is motivated by the conflict between respect for, and scepticism about, models for economic forecasting. As William Poole put it (2004):

> The true art of good monetary policy is in managing forecast surprises and not in doing the obvious things implied by the baseline forecast. (p.1) ... [P]olicy needs to be informed by the best guesses incorporated in forecasts and by knowledge of forecast errors. Forecast errors create risk, and that risk needs to be managed as efficiently as possible. (p.5)

This paper presents a specific methodological response to Poole's perspective, based on info-gap theory which is a quantitative methodology for analysis and design of policy with severe uncertainty — surprises.

The following implications of info-gap decision theory are developed in this paper:

1. Do not attempt to exhaustively list adverse events. Surprises by their nature cannot be anticipated.
2. While we cannot forecast surprises, info-gap theory enables one to model one's ignorance of those surprises.
3. Info-gap theory is not a worst-case analysis. While there may be a worst case, one cannot know what it is and one should not base one's policy upon guesses of what it might be. As such, info-gap theory is different from robust-control and min-max methods. The strategy advocated here is *not* the amelioration of purportedly worst cases.
4. The basic tool of info-gap policy formulation is a quantitative answer to the **robustness question**: For a specified policy, how wrong can our models and data be, without jeopardizing the outcome of that policy? The answer is provided by the info-gap robustness function. The difference from min-max approaches is that we are able to select a policy without ever specifying how wrong the model actually is.
5. A policy which is robust to surprises is preferable to a vulnerable policy.
6. Highly ambitious policy is more vulnerable to surprises than a policy aimed at modest goals. That is, policy goals trade-off against immunity to uncertainty.
7. Optimization of policy goals (e.g., minimizing output gap or maximizing inflation stability) is equivalent to minimizing the immunity to uncertainty.
8. Consequently, policy goals should be 'good enough' but not necessarily optimal, in order to obtain robustness against surprises. That is, policy should be chosen to **satisfice** the goals and not to optimize them.
9. Goals which are satisficed (sub-optimal but good enough) can be achieved by many alternative policies. Choose the most robust from among these alternatives.
10. We identify situations in which the robust-satisficing and optimizing strategies are the same.

In summary, info-gap theory provides a quantitative tool for policy formulation and evaluation which is based on Knight's uncertainty and Simon's bounded rationality. We cannot predict surprises, but info-gap theory enables us to model and manage our ignorance of those surprises. Info-gap policy formulation is particularly suited to situations in which surprises are critically important.

We begin with a simple heuristic example in section 2 through which we introduce the method of info-gap robust-satisficing. We then proceed to a slightly more involved example: the choice of a Taylor rule for controlling inflation, output gap and interest rate fluctuations in section 3. The economic model is described in section 3.1. Info-gap models of uncertainty are defined and discussed in section 3.2. The main tool of info-gap analysis is the robustness function, which is developed in section 3.4 and applied to policy selection in section 3.6. Section 4 contains a brief methodological summary.

2 Preliminary Example

We will consider an adaptation of Brainard's (1967) example which is discussed by Blinder (1998, pp. 11–12). We will examine policy selection for this example from an info-gap perspective with severe Knightian uncertainty. We will show that policies which, based on best-estimated models would seem to optimize the outcome, should sometimes be avoided, as was suggested by Brainard.

Consider the macroeconomic model:

$$y = Gx + z \tag{1}$$

where G and z are both highly uncertain, with best-estimates $\widetilde{G}$ and $\widetilde{z}$, respectively.

We have no probabilistic model for the error in the estimates[1] $\widetilde{G}$ and $\widetilde{z}$, and what we can say is that the fractional error in these estimates is unknown. That is, true (or truer) values G and z deviate from the estimated values $\widetilde{G}$ and $\widetilde{z}$ by no more than a fraction α. However, this 'horizon of uncertainty' α is unknown. An info-gap model for this uncertainty is the following unbounded family of nested sets of G and z values:

$$\mathcal{U}(\alpha, \widetilde{G}, \widetilde{z}) = \left\{ G, z : \ \left| \frac{G - \widetilde{G}}{\widetilde{G}} \right| \le \alpha, \ \left| \frac{z - \widetilde{z}}{\widetilde{z}} \right| \le \alpha \right\}, \quad \alpha \ge 0 \tag{2}$$

At any horizon of uncertainty, α, the estimates $\widetilde{G}$ and $\widetilde{z}$ may err fractionally by as much as α. However, the value of α is not known. Thus an info-gap model does not allow a 'worst case' analysis: these is no known worst case since the horizon of error is unknown. We are deep in the domain of Knightian uncertainty.

The performance function is the squared difference between the desired value $y^\star$ and the realized value y:

$$f(x, G, z) = [y(x, G, z) - y^\star]^2 \tag{3}$$

In the spirit of Simon's bounded rationality and the concept of satisficing, we desire the performance function to be no greater than the critical value $E_{\rm c}^2$:

$$f(x, G, z) \le E_{\rm c}^2 \tag{4}$$

$E_{\rm c}^2$ can be chosen to be small or large to express demanding or modest performance aspirations.

The **robustness** of policy choice x is the greatest fractional error in the estimates $\widetilde{G}$ and $\widetilde{z}$, up to which every realization G and z results in acceptable squared error. Formally, the robustness of decision x with aspiration $E_{\rm c}$ is:

$$\widehat{\alpha}(x, E_{\rm c}) = \max \left\{ \alpha : \ \left(\max_{G, z \in \mathcal{U}(\alpha, \widetilde{G}, \widetilde{z})} f(x, G, z) \right) \le E_{\rm c}^2 \right\} \tag{5}$$

Large robustness $\widehat{\alpha}(x, E_{\rm c})$ implies that policy choice x is immune to error in the estimates while satisficing the outcome-error at $E_{\rm c}$. Low robustness implies that outcome-error as small as $E_{\rm c}$ cannot be confidently expected with choice x.

The robustness function induces a preference ordering on the choice variable: x is preferred over x' if the former is more robust than the latter, at the same aspiration $E_{\rm c}$. Formally, the robust-satisficing preference relation at aspiration $E_{\rm c}$ is:

$$x \succ x' \quad \text{if} \quad \widehat{\alpha}(x, E_{\rm c}) > \widehat{\alpha}(x', E_{\rm c}) \tag{6}$$

[1] Info-gap theory can be used very fruitfully in dealing with Knightian uncertainty in probability models, e.g., unknown errors in the tails of a pdf. This 'hybrid uncertainty' is not developed here.

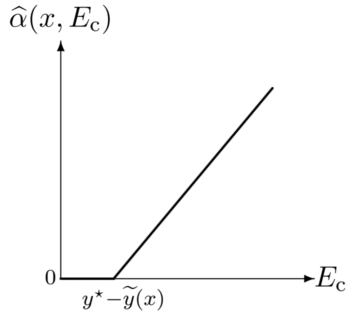

Figure 1: Schematic illustration of the robustness function $\widehat{\alpha}(x, E_c)$ in eq.(7).

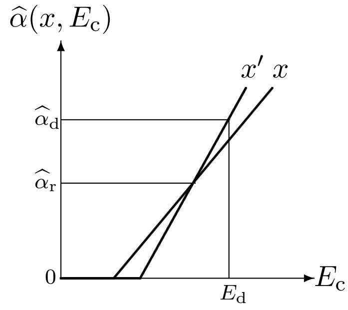

Figure 2: Comparison of two policy choices: reversal of preferences.

Let $\widetilde{y}(x) = \widetilde{G}x + \widetilde{z}$ denote the best estimate of the outcome, given choice x, and let us consider values of x so that $\widetilde{y}(x) \le y^\star$. We will assume that $\widetilde{G} > 0$ and $\widetilde{z} > 0$. The robustness of choice x is found to be:

$$\widehat{\alpha}(x, E_c) \begin{cases} \dfrac{E_c - [y^\star - \widetilde{y}(x)]}{\widetilde{y}(x)} & \text{if } E_c \ge y^\star - \widetilde{y}(x) \\ 0 & \text{else} \end{cases} \tag{7}$$

Outcome-error no greater than E_c is guaranteed with policy choice x if the horizon of uncertainty is no larger than $\widehat{\alpha}(x, E_c)$.

As illustrated in fig. 1, the robustness increases (gets better) as the aspired fidelity E_c increases (gets worse). This illustrates a general theorem of info-gap theory: robustness trades-off against performance.

Furthermore we see in eq.(7) and fig. 1 that the robustness vanishes when the aspiration E_c equals the best-estimate of the fidelity, $y^\star - \widetilde{y}(x)$. This is true for any choice of x. We can have little confidence in attaining fidelity as good as the best-estimated fidelity; only poorer fidelity has positive robustness. Since this is true for any x, it is also true for the choice of x which minimizes the estimated error, $f(x, \widetilde{G}, \widetilde{z})$.

This is beginning to sound like Brainard's conclusion that policies which optimize the outcome should sometimes be avoided, but there is more.

Let us now consider two policy alternatives, x and x', where:

$$\widetilde{y}(x) > \widetilde{y}(x') \tag{8}$$

In particular, let x be the policy choice which, based on the best-estimated model, causes the outcome to precisely match the required value: $\widetilde{y}(x) = y^\star$. This choice of x is what would normally be called the optimal policy. Eq.(8) means that the estimated fidelity is worse with x' than with x:

$$0 = y^\star - \widetilde{y}(x) < y^\star - \widetilde{y}(x') \tag{9}$$

The robustness curves for choices x and x' are shown in fig. 2. Since the best-estimated fidelity of x' is poorer than for x, eq.(9), $\widehat{\alpha}(x', E_c)$ intersects the E_c-axis to the right of $\widehat{\alpha}(x, E_c)$. However, eqs.(7) and (8) imply that the slope of $\widehat{\alpha}(x', E_c)$ is steeper than the slope of $\widehat{\alpha}(x, E_c)$, so these robustness curves cross.

Crossing of robustness curves implies reversal of preference between choices x and x'. Let us suppose that the value of robustness, $\widehat{\alpha}_r$, at which these curves cross is fairly low. If we are quite confident that the estimates $\widetilde{G}$ and $\widetilde{z}$ are accurate, then we don't need much robustness, so $\widehat{\alpha}_r$ might be enough robustness and we would prefer choice x over choice x' based on the preference relation in eq.(6). However, we are considering severe Knightian uncertainty: great error in $\widetilde{G}$ and $\widetilde{z}$ is plausible and we need to choose a policy whose anticipated outcome is both acceptable and reliably achieved. Thus, if outcome-error $E_{\rm d}$ is good enough fidelity, and if $\widehat{\alpha}_{\rm d}$ is great enough robustness, then our robust-satisficing preference, eq.(6), is for x' over x. If an acceptable combination ($E_{\rm d}$, $\widehat{\alpha}_{\rm d}$) is not found on the x'-curve, then we need to search for some other choice, x'', whose robustness is adequate at acceptable fidelity. If no such x'' exists, then no acceptable policy choice is available in the current state of knowledge. We either revise our aspirations, or do some data-hunting, or revise our model.

This very simple example has illustrated the info-gap robust-satisficing motivation for Brainard's dictum — calculate the optimum and then do less — or for what Blinder refers to as "a little stodginess at the central bank" (Blinder 1998, p.12). Fig. 2 shows that the optimal choice, x, is less desirable than the sub-optimal choice x' under severe uncertainty, because the latter more reliably yields acceptable outcomes (if $E_{\rm d}$ is adequate).

While crossing of robustness curves as in this example is very common, it is not universal. It can happen that robustness curves do not cross, in which case the policy-selection stodginess disappears: the optimizing choice will coincide with the robust-satisficing choice. However, the caution remains in assessing what outcome can be considered reliable. Since the trade-off between robustness and outcomes is universal, the robust-satisficing policy maker will not anticipate (or depend upon) the best-estimated outcome because the robustness of this outcome is zero. Rather, by "migrating up" the robustness curve to an acceptable level of robustness, the analyst finds the corresponding outcome which can reliably be anticipated.

3 Choosing Taylor Coefficients

3.1 The Economic Model

We use the following model for inflation and output gap, based on Rudebusch and Svensson (1999) (see also Onatski and Stock (2000)):

$$\pi_{t+1} = a_0\pi_t + a_1\pi_{t-1} + a_2\pi_{t-2} + a_3\pi_{t-3} + by_t \qquad (10)$$

$$y_{t+1} = c_0 y_t + c_1 y_{t-1} + d(\bar{\imath}_t - \overline{\pi}_t) \qquad (11)$$

t is the time step in quarters. π_t is the deviation of the inflation from a target value (or the inflation itself), in the tth quarter. y_t is the output gap at time t, measured as 100 times the log ratio of the actual real output to the potential output. i_t is the Federal funds rate at an annual rate, and $\bar{\imath}_t$ is the 4-quarter average Federal funds rate:

$$\bar{\imath}_t = 0.25(i_t + i_{t-1} + i_{t-2} + i_{t-3}) \qquad (12)$$

Likewise, $\overline{\pi}_t$ is the 4-quarter average of the inflation variable:

$$\overline{\pi}_t = 0.25(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}) \qquad (13)$$

The Federal funds rate is regulated by a Taylor rule:

$$i_t = g_\pi \overline{\pi}_t + g_y y_t \qquad (14)$$

where g_π and g_y are decision variables to be chosen by the policy maker.

The original model of Rudebusch and Svensson includes zero-mean shocks $\varepsilon_{\pi,t+1}$ and $\varepsilon_{y,t+1}$ in eqs.(10) and (11) respectively. We model shocks and surprises by considering uncertainty in the coefficients of the model, as discussed in section 3.2.

The eight coefficients in eqs.(10) and (11) have been estimated by Rudebusch and Svensson (1999). Their values, with standard errors, are shown in table 1.

	a_0	a_1	a_2	a_3	b	c_0	c_1	d
Mean	0.07	−0.10	0.28	0.12	0.14	1.16	−0.25	−0.10
Standard Error	0.08	0.10	0.10	0.08	0.03	0.08	0.08	0.03

Table 1: Mean and standard error of coefficients in eqs.(10) and (11).

We adopt the following notation for time-sequences of variables:

$$\pi_{t(k)} = (\pi_t, , \ldots, \pi_{t-k})^T \tag{15}$$

$$y_{t(k)} = (y_t, , \ldots, y_{t-k})^T \tag{16}$$

Furthermore, let $F = (a_0, \ldots, a_3, b, c_0, c_1, d)$ denote the 8 model coefficients (which we subsequently will consider to be uncertain).

Eqs.(10)–(14) constitute the economic model. Examination of these equations shows that, given values of $\pi_{t(6)}$ and $y_{t(3)}$, the economic model predicts the deviation of the inflation, π_{t+1}, and output gap y_{t+1}. This one-step prediction is bilinear in F and $(\pi_{t(6)}, y_{t(3)})$. We denote these 1-step average predictions by $\pi_{t+1}[F|\pi_{t(6)}, y_{t(3)}]$ and $y_{t+1}[F|\pi_{t(6)}, y_{t(3)}]$, respectively.

3.2 Model Uncertainty

In this section we formulate and discuss an info-gap model for uncertainty in the coefficients F of the economic model, eqs.(10) and (11). The quantitative information which we have about these 8 coefficients is their empirically estimated means and standard deviations, shown in table 1, which we denote as $\widetilde{F}_k$ and s_k, $k = 1, \ldots, 8$. The basic Knightian intuition is that unknown future surprises cannot be insured against and cannot be modelled probabilistically because the rules governing future surprises are unknowable based on past experience (Knight 1921). We anticipate that more accurate values F_k will deviate from the estimates $\widetilde{F}_k$, and that the relative tendencies for deviation of the coefficients are expressed by the standard errors s_k. However, the actual magnitude of the error of the estimates is unknown and we have no probability measure of these errors. We therefore adopt the following unbounded-interval-uncertainty info-gap model (Ben-Haim 2006):

$$\mathcal{U}(\alpha, \widetilde{F}) = \left\{ F : \ \frac{|F_k - \widetilde{F}_k|}{s_k} \le \alpha, \ k = 1, \ldots, 8 \right\}, \quad \alpha \ge 0 \tag{17}$$

This is an unbounded family of nested sets of coefficient-intervals. For any given value of α, the set $\mathcal{U}(\alpha, \widetilde{F})$ defines a range of variation of the model coefficients. However, the 'horizon of uncertainty' α is unknown, so the info-gap model contains an unbounded family of nested sets of possible realizations of the coefficients F_k. For further discussion of the relation between Knightian uncertainty and info-gap models see Ben-Haim (2004, 2006).

It is important to emphasize that an info-gap model is *not* a realization of "bounded uncertainty". This is important for two reasons, one methodological and one epistemic. First, there is (usually) no worst case in an info-gap model, since the horizon of uncertainty is unknown and unbounded. Methodologically, this means that an info-gap analysis is fundamentally different from 'worst case' or 'min-max' analysis. Second, the assertion of a sharp boundary which delineates 'possible' from 'impossible' is, epistemically, a very strong assertion. Sharp bounds are usually difficult to verify unless they are in effect not meaningful. (An illustration of a true but meaningless sharp bound: 'The log-ratio of the output gap is bounded by 10^8'.) The verification of a meaningful sharp cutoff of a highly uncertain quantity is empirically difficult, and should not be done incautiously. If such an assertion *is* verified empirically, then quite possibly a much more informative uncertainty model (e.g., a probability model) can be verified and should be used.

3.3 Aspirations

The policy maker wants to choose the coefficients of the Taylor rule, $g^T = (g_\pi,\ g_y)$, to achieve acceptably small inflation-target deviation $|\pi_t|$, output gap $|y_t|$, and interest rate fluctuation $|i_{t+1} - i_t|$, over a specified time horizon $t = 1, \ldots, T$. Since the coefficients of the model are highly uncertain and subject to unanticipated variation, it is not possible to reliably predict the outcome of any specific choice of the Taylor coefficients g. However, it is possible to evaluate a proposed g in terms of how robust, to uncertainty in the model, the resulting behavior is. We do this with the info-gap robustness function, which we now formulate.

Small values of $|\pi_t|$, $|y_t|$ and $|i_t - i_{t-1}|$, over the time horizon $t = 1, \ldots, T$, are desired. It is unlikely that these quantities will become identically zero. Hence the policy maker specifies tolerances (which we will evaluate later) on the achievement of these targets. The policy, g, is considered acceptable if:

$$|\pi_t| \leq r_{\pi,t}, \quad t = 1, \ldots, T \tag{18}$$

$$|y_t| \leq r_{y,t}, \quad t = 1, \ldots, T \tag{19}$$

$$|i_t - i_{t-1}| \leq r_{i,t}, \quad t = 2, \ldots, T \tag{20}$$

Let r_π, r_y and r_i denote the vectors of tolerances. By specifying tolerances such as these, the policy maker aspires to **satisfice** the dynamic variables of the system, rather than to minimize or optimize them.

The basic robustness question is: how wrong can the estimated model, $\widetilde{F}$, be, without violating the policy maker's aspirations expressed in eqs.(18)–(20)? This is not a min-max analysis, and we do *not* ask the question: what *is* the greatest horizon of uncertainty which can or will be observed?

We now construct the info-gap answer to this question. We begin by constructing robustness functions for each of the three variables, π, y and i. We then examine the performance-vs.-aspiration trade-off expressed by the robustness functions. We then demonstrate the trade-off between robustifying the output gap and robustifying the interest-rate increment. Finally, we show that the estimated best performance has no immunity to model uncertainty.

3.4 Robustness Functions

First consider the inflation variable. The robustness, to model uncertainty, of Taylor coefficients g, given inflation-aspirations r_π, is the greatest horizon of uncertainty α at which the aspirations are not violated by any economic model F in $\mathcal{U}(\alpha, \widetilde{f})$, throughout time horizon T:

$$\widehat{\alpha}_\pi(g, r_\pi, T) = \max\left\{\alpha : \left(\max_{F \in \mathcal{U}(\alpha, \widetilde{F})} |\pi_t|\right) \leq r_{\pi,t}, \quad \forall \ \ t = 1, \ldots, T\right\} \tag{21}$$

A large value of $\widehat{\alpha}_\pi(g, r_\pi, T)$ is desirable, and means that Taylor coefficients g can be relied upon to keep the inflation-deviations $|\pi_t|$ within the specified tolerances r_π. A small value of the robustness implies that the policy cannot be relied upon to achieve the aspirations. Clearly "bigger is better" for robustness, so the robust-optimal Taylor coefficients, regarding inflation, are those which maximize the robustness while satisficing the inflation performance:

$$\widehat{g}_\pi(r_\pi) = \arg\max_g \widehat{\alpha}_\pi(g, r_\pi, T) \tag{22}$$

The robustness of Taylor coefficient g for output gap aspirations and, separately, for interest rate stability, are formulated in a similar manner:

$$\widehat{\alpha}_y(g, r_y, T) = \max\left\{\alpha : \left(\max_{F \in \mathcal{U}(\alpha, \widetilde{F})} |y_t|\right) \leq r_{y,t}, \quad \forall \ \ t = 1, \ldots, T\right\} \tag{23}$$

$$\widehat{\alpha}_i(g, r_i, T) = \max\left\{\alpha : \left(\max_{F \in \mathcal{U}(\alpha, \widetilde{F})} |i_t - i_{t-1}|\right) \leq r_{i,t}, \quad \forall \ \ t = 1, \ldots, T\right\} \tag{24}$$

The robust-optimal Taylor coefficients for output gap and interest stability are, respectively:

$$\widehat{g}_y(r_y) = \arg\max_g \widehat{\alpha}_y(g, r_y, T) \quad (25)$$

$$\widehat{g}_i(r_i) = \arg\max_g \widehat{\alpha}_i(g, r_i, T) \quad (26)$$

The robustness for satisficing the aspirations regarding all three variables is the smallest of the three robustness:

$$\widehat{\alpha}(g, r_\pi, r_y, r_i,\ T) = \min\left\{\widehat{\alpha}_\pi(g, r_\pi, T),\ \widehat{\alpha}_y(g, r_y, T),\ \widehat{\alpha}_i(g, r_i, T)\right\} \quad (27)$$

The corresponding robust-optimal Taylor coefficients maximize this robustness:

$$\widehat{g}(r_\pi, r_y, r_i) = \arg\max_g \widehat{\alpha}(g, r_\pi, r_y, r_i, T) \quad (28)$$

3.5 Trade-off of Performance Against Robustness

It is worthwhile to make three brief general observations before proceeding to a numerical example. For convenience, I will refer generically to any of the three dynamic variables, π, y and i, by x, and denote the corresponding robustness functions by $\widehat{\alpha}_x(g, r_x, T)$.

Trade-off of robustness and performance. It is readily demonstrated (Ben-Haim 2006) that the robustness improves as the aspiration becomes more modest:

$$r_x < r'_x \quad \text{implies} \quad \widehat{\alpha}_x(g, r_x, T) \le \widehat{\alpha}_x(g, r'_x, T) \quad (29)$$

$r_x < r'_x$ means that $r_{x,t} < r'_{x,t}$ for at least one value of t up to T. r_x is the vector of levels at which the policy maker aspires to satisfice the dynamic variable: x_t must not exceed $r_{x,t}$ at each t up to T. Large values of r_x imply modest aspiration; small values are demanding. That the robustness increases as the aspiration falls is a direct result of the nesting of the info-gap model of uncertainty.

Performance-optimization is undependable. Onatski and Williams note that "optimal rules may perform poorly when faced with a different shock distribution, or slight variation in the model." (Onatski and Williams 2003). This observation is supported by the info-gap robustness analysis, as we now explain.

Let $\widetilde{x}$ denote the vector of values of x in each quarter up to T, based on the estimated coefficients, $\widetilde{F}$. It can be proven that the outcome $\widetilde{x}$ anticipated from the estimated model $\widetilde{F}$ cannot be relied upon, since its robustness is zero:

$$\widehat{\alpha}_x(g, r_x, T) = 0 \quad \text{if} \quad r_x = \widetilde{x} \quad (30)$$

This relation holds for any choice of the Taylor coefficients g. In particular, the robustness to model uncertainty is zero if g is chosen to optimize (e.g. minimize) the outcomes x. That is, if g is chosen so that $\widetilde{x}$ are optimal performance outcomes based on the estimated model $\widetilde{F}$, then the robustness to model uncertainty is zero for achieving these optimal outcomes. This motivates the adoption of the performance-satisficing strategy which is advocated here. Performance-optimization based on the estimated model has no immunity to error, so it is fatuous to choose the Taylor coefficients g, based on $\widetilde{F}$, to optimize the economic variables. Positive robustness to model uncertainty can be obtained only by accepting sub-optimal performance. We will explore an example in section 3.6. (The proof of a general theorem, of which relation (30) is a special case, appears in (Ben-Haim 2005)).

Preference reversal. Robustness curves can cross, as we have already seen in fig. 2. The crossing of robustness curves means that, at some level of performance, a policy which is sub-optimal according to the best model, will be more robust than a best-model optimum policy. Since positive robustness is desirable, this can induce a preference (under severe uncertainty) for the sub-optimal policy (which would not be preferred under low uncertainty).

3.6 Policy Analysis: 1-Quarter Time Horizon

In this section we consider the robustness for inflation gap π_{t+1}, output gap y_{t+1}, and interest rate fluctuation $i_{t+1} - i_t$, for a 1-quarter time horizon. The purpose is to illustrate in detail the procedure for evaluating the info-gap robustness functions, and to demonstrate their meaning. We concentrate on the info-gap uncertainty in the model coefficients. We explore the implications for choice of the Taylor coefficients. We will find that, for a 1-quarter time horizon, $\widehat{\alpha}_\pi$ is independent of g, $\widehat{\alpha}_y$ improves as either g_π or g_y increases, and $\widehat{\alpha}_i$ deteriorates as $\widehat{\alpha}_y$ improves. All of the robustnesses are very low for the range of g values considered. Robustnesses for a longer time horizon will be even lower.

3.6.1 Derivation of Robustness Functions

Inflation gap. First consider the robustness to model-uncertainty of the deviation of the inflation from its target value, $\widehat{\alpha}_\pi(g, r_\pi, 1)$ in eq.(21). The projected inflation-deviation, one quarter ahead, is:

$$\pi_{t+1}[F|\pi_{t(6)}, y_{t(3)}] = by_t + \sum_{k=0}^{3} a_k \pi_{t-k} \tag{31}$$

Using the intervals of unknown size in the info-gap model in eq.(17), one finds that the maximal absolute value of π_{t+1}, up to horizon of uncertainty α, is:

$$\max_{F \in \mathcal{U}(\alpha, \widetilde{F})} \left| \pi_{t+1}[F|\pi_{t(6)}, y_{t(3)}] \right| = \underbrace{\left| \widetilde{b} y_t + \sum_{k=0}^{3} \widetilde{a}_k \pi_{t-k} \right|}_{|\widetilde{\pi}_{t+1}|} + \alpha \underbrace{\left[|y_t| s_b + \sum_{k=0}^{3} |\pi_{t-k}| s_{a_k} \right]}_{\zeta} \tag{32}$$

where $\widetilde{a}_k$ and $\widetilde{b}$ are the best estimates: the centerpoint $\widetilde{F}$ of the info-gap model, $\widetilde{\pi}_{t+1}$ is the anticipated inflation gap in the next quarter, and ζ arises from the info-gap model and depends only on past state variables.

The robustness of the inflation gap is found by equating the righthand side of eq.(32) to the critical value r_π and solving for α:

$$\widehat{\alpha}_\pi(g, r_\pi, 1) = \begin{cases} 0 & \text{if } |\widetilde{\pi}_{t+1}| \geq r_\pi \\ \dfrac{r_\pi - |\widetilde{\pi}_{t+1}|}{\zeta} & \text{else} \end{cases} \tag{33}$$

Note that the inflation-gap robustness for a 1-quarter time horizon does not depend on the coefficients, g, of the Taylor relation.

Output gap. The one-step projection of the output gap is:

$$y_{t+1}[F|\pi_{t(6)},\, y_{t(3)}] = c_0 y_t + c_1 y_{t-1} + d \underbrace{\left(\frac{g_y}{4} \sum_{k=0}^{3} y_{t-k} + \frac{g_\pi}{16} \sum_{k=0}^{6} \nu_k \pi_{t-k} - \frac{1}{4} \sum_{k=0}^{3} \pi_{t-k} \right)}_{A} \tag{34}$$

where $\nu^T = (1,\, 2,\, 3,\, 4,\, 3,\, 2,\, 1)$. The maximum absolute inflation gap, up to uncertainty α, is:

$$\max_{F \in \mathcal{U}(\alpha, \widetilde{F})} \left| y_{t+1}[F|\pi_{t(6)}, y_{t(3)}] \right| = \underbrace{\left| \widetilde{c}_0 y_t + \widetilde{c}_1 y_{t-1} + \widetilde{d} A \right|}_{|\widetilde{y}_{t+1}|} + \alpha \underbrace{\left(s_{c_0} |y_t| + s_{c_1} |y_{t-1}| + s_d |A| \right)}_{\theta} \tag{35}$$

where $\widetilde{c}_0$, $\widetilde{c}_1$ and $\widetilde{d}$ are the best-estimates, $\widetilde{y}_{t+1}$ is the anticipated projected output gap, and θ results from the info-gap model and depends only on past state variables. The robustness of the output gap is evaluated by equating this expression to the critical output gap and solving the α:

$$\widehat{\alpha}_y(g, r_y, 1) = \begin{cases} 0 & \text{if } |\widetilde{y}_{t+1}| \geq r_y \\ \dfrac{r_y - |\widetilde{y}_{t+1}|}{\theta} & \text{else} \end{cases} \tag{36}$$

Note that the the coefficients, g, of the Taylor relation appear in the output-gap robustness for a 1-quarter time horizon (though θ and A). This is unlike the inflation-gap robustness.

Interest rate fluctuation. Eqs.(10)–(14) can be combined to expressed the 1-quarter increment in the interest rate as:

$$i_{t+1} - i_t = \underbrace{-g_y y_t - \frac{g_\pi}{4}\pi_{t-3}}_{B} + \frac{g_\pi}{4}\left(b y_t + \sum_{k=0}^{3} a_k \pi_{t-k}\right) + g_y\left(c_0 y_t + c_1 y_{t-1} + dA\right) \tag{37}$$

where A is defined in eq.(34). The maximum absolute value of the 1-quarter interest rate increment, up to horizon of uncertainty α, is:

$$\max_{F \in \mathcal{U}(\alpha, \widetilde{F})} |i_{t+1} - i_t| = \left| B + \frac{g_\pi}{4}\widetilde{\pi}_{t+1} + g_y \widetilde{y}_{t+1} \right| + \alpha\omega \tag{38}$$

where $\widetilde{\pi}_{t+1}$ is defined in eq.(32), $\widetilde{y}_{t+1}$ is defined in eq.(35), and where we have defined:

$$\omega = \frac{s_b}{4}|g_\pi y_t| + \frac{1}{4}\sum_{k=0}^{3} s_{a_k}|g_\pi \pi_{t-k}| + s_{c_0}|g_y y_t| + s_{c_1}|g_y y_{t-1}| + s_d|g_y A| \tag{39}$$

The robustness of the interest-rate increment, to uncertainty in the model coefficients, is found by equating the righthand side of eq.(38) to r_i and solving for α:

$$\widehat{\alpha}_i(g, r_i, 1) = \begin{cases} 0 & \text{if } \left| B + \frac{g_\pi}{4}\widetilde{\pi}_{t+1} + g_y \widetilde{y}_{t+1} \right| \geq r_i \\ \dfrac{r_i - \left| B + \frac{g_\pi}{4}\widetilde{\pi}_{t+1} + g_y \widetilde{y}_{t+1} \right|}{\omega} & \text{else} \end{cases} \tag{40}$$

3.6.2 Results and Discussion

We begin by discussing the robustness, to model uncertainty, of the inflation-gap and the output-gap. Eqs.(33) and (36) illustrate the trade-off of aspiration against robustness to model uncertainty discussed in eq.(29): both $\widehat{\alpha}_\pi(g, r_\pi, 1)$ and $\widehat{\alpha}_y(g, r_y, 1)$ increase (indicating greater robustness) as r_π or r_y increase (indicating more modest inflation- or output-aspiration). This is illustrated in fig. 3.

The inflation-gap robustness $\widehat{\alpha}_\pi(g, r_\pi, 1)$ exceeds the output-gap robustness $\widehat{\alpha}_y(g, r_y, 1)$ over the range of r_π and r_y values shown, with Taylor coefficients $g_\pi = 1.5$ and $g_y = 0.5$ (which are the values suggested by Taylor (Onatski and Stock 2000, p.6)). From fig. 3 we see that $\widehat{\alpha}_\pi(g, r_\pi, 1)$ reaches a value of 1.4 at $r_\pi = 2.5$. This means that each model coefficient F_k can vary by $\pm 1.4 s_k$ around its best estimate, $\widetilde{F}_k$, without causing the 1-step inflation to deviate from the target value by more than 2.5 percentage points. Deviation of the inflation target by 2.5 percentage points is quite significant but much within the 'tradition' of forecast surprises as discussed by Poole (2004). The info-gap robustness predicts that such large deviations can occur even though the model coefficients err by no more than 1.4 standard deviations. The point is that the info-gap robustness function $\widehat{\alpha}_\pi(g, r_\pi, 1)$ is providing a fairly realistic assessment of the vulnerability to surprises. Furthermore, $\widehat{\alpha}_\pi(g, r_\pi, 1)$ incorporates a specific policy option into this surprise-modelling.

The inflation-gap robustness is zero for $r_\pi \leq 1.9$. This means that 1-quarter inflation target deviation of 1.9% cannot be reliably predicted with this model: arbitrarily small error in the model

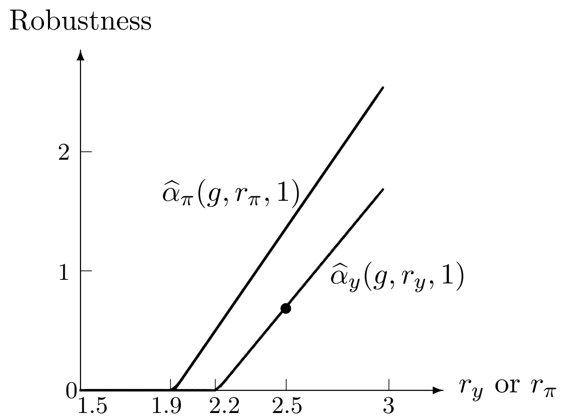

Figure 3: Robustness of output and inflation gaps vs. r_y and r_π. $g_\pi = 1.5$, $g_y = 0.5$, $\pi_{t(6)} = $ (1.5, 0.32, 2.2, −0.35, −1.6, 1.8, 2.9), $y_{t(3)} = $ (2.6, 3.1, −0.8, 1.7).

coefficients can cause the inflation to deviate from its target value by more than 1.9%. Consequently, this economic model, eqs.(10) and (11), cannot be reliably used to argue that Taylor coefficients $g_\pi = 1.5$ and $g_y = 0.5$ will yield 1-quarter inflation within 1.9% of the target value. This is significant since, as discussion of eq.(30) indicates, the estimated model $\widetilde{F}$ predicts a 1-quarter inflation gap of 1.9%. We see that this is a very unreliable prediction since $\widehat{\alpha}_\pi(g, 1.9, 1) = 0$.

The inflation-gap robustness, $\widehat{\alpha}_\pi$, reaches a value of 2.6 at $r_\pi = 3$, indicating that ±3 percentage points of inflation-gap can be fairly well depended on, with the Taylor coefficients used here. Note that these specific numerical results depend on the inflation-gap and output-gap history, $\pi_{t(6)}$ and $y_{t(3)}$, specified in the caption.

Still referring to fig. 3, the output-gap robustness, $\widehat{\alpha}_y(g, r_y, 1)$, is consistently below the inflation-gap robustness $\widehat{\alpha}_\pi(g, r_\pi, 1)$ by about 0.6 to 0.8. In other words this choice of the Taylor coefficients yields substantially less output-gap robustness to model uncertainty. For example $\widehat{\alpha}_y(g, 2.5, 1) = 0.70$ (compared with $\widehat{\alpha}_\pi(g, 2.5, 1) = 1.4$) and $\widehat{\alpha}_y(g, 3, 1) = 1.7$ (compared with $\widehat{\alpha}_\pi(g, 3, 1) = 2.6$). A robustness of $\widehat{\alpha}_y(g, 2.5, 1) = 0.70$ means that each model coefficient, F_k, can vary by $\pm 0.70 s_k$, around its estimated value $\widetilde{F}_k$, without causing the output gap to exceed 2.5. (Recall that the output gap is measured as 100 times the logarithm of the ratio of actual real output to potential output. Hence an output-gap aspiration of $r_y = 2.5$ corresponds to an actual-to-potential output ratio between 0.94 and 1.06.) Once again we see that very modest modelling error — 0.7 standard deviations — results in economically very significant prediction error.

Now consider the robustness of the interest rate increment, $\widehat{\alpha}_i(g, r_i, 1)$, as shown in fig. 4. Robustness curves are shown for three choices of the Taylor coefficients. As we know from eq.(40), the slope is positive which expresses the trade-off between robustness, to model uncertainty, and the aspired limit on the interest rate change, r_i.

Let $\Delta\widetilde{i}$ denote the estimate of the interest-rate change, $|i_t - i_{t-1}|$, based on the best-estimates of the model parameters, $\widetilde{F}$. $\Delta\widetilde{i} = 0.75$, 0.65 and 0.47 for $g_y = 0.296$, 0.492 and 0.811 respectively. Choosing $g_y = 0.811$ has the best performance, from among the three values displayed, since it has the lowest best-model interest-rate change: 0.47. However, the robustness is precisely zero for the best-model prediction: choosing $r_i = \Delta\widetilde{i}$ causes $\widehat{\alpha}_i = 0$ as seen in the figure. Only greater values of r_i have positive robustness. Furthermore, since the robustness curves cross at about a robustness of $\widehat{\alpha} = 1$, we see that we prefer $g_y = 0.296$ if robustness to more than one standard deviation of the parameters is needed.

In short, fig. 4 demonstrates **reversal of preferences:** Taylor coefficients which are indicated by the best-estimated model are not selected when the robustness to model-uncertainty is considered.

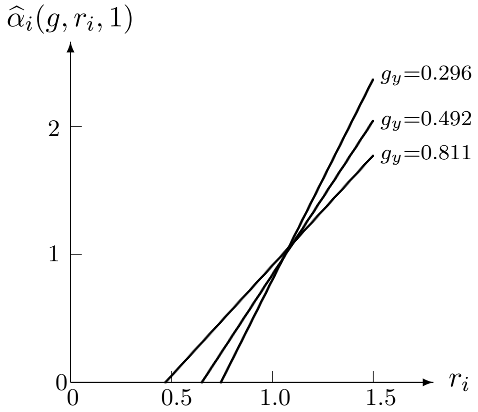

Figure 4: Robustness of interest rate increment vs. r_i, for $g_\pi = 1.5$ and for $g_y = 0.296$, 0.492 and 0.811. $\pi_{t(6)}$ and $y_{t(3)}$ same as in fig. 3.

Now consider the variation of the robustness with the Taylor coefficients g. As noted following eq.(33), the inflation-gap robustness with a 1-quarter time horizon is actually independent of the Taylor coefficients. The output-gap robustness does, however, depend on g, as shown in fig. 5.

The dot in fig. 5 corresponds to the same constellation of parameters as the dot in fig. 3: $r_y = 2.5$, $g_y = 0.5$, $g_\pi = 1.5$. It is evident from fig. 5 that substantially improved robustness can be obtained by adjusting the Taylor coefficients. The robustness at the dot is $\widehat{\alpha}_y = 0.70$, while the greatest robustness in fig. 5 is $\widehat{\alpha}_y = 1.6$, occuring at $g_y = 5$ and $g_\pi = 4.5$. A unit increment in robustness indicates that each model coefficient F_k can vary by an additional increment of s_k around its estimated value $\widetilde{F}_k$, without jeopardizing the output-gap aspiration.

The next thing to note from fig. 5 is that g_y influences the robustness substantially more than g_π does. At fixed g_y, the robustness $\widehat{\alpha}_y(g, r_y, 1)$ varies by 0.2 to 0.4, over the range of g_π values. This is quite a bit less than the influence of varying g_y at fixed g_π: $\widehat{\alpha}_y(g, r_y, 1)$ changes by 0.8 to 1.0.

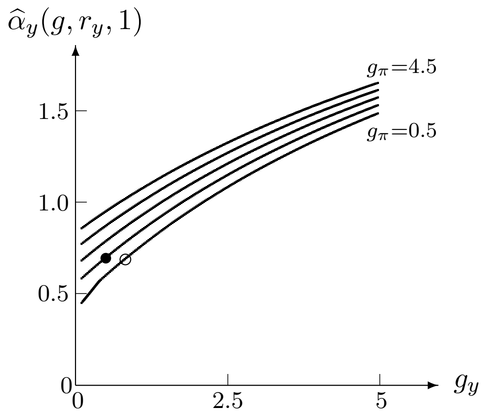

Figure 5: Robustness of output gap increment vs. g_y, for $g_\pi = 0.5, 1.5, \ldots,\ 4.5$ (bottom to top). $\pi_{t(6)}$ and $y_{t(3)}$ same as in fig. 3. $r_y = 2.5$.

Having concluded from our analysis of fig. 5 that the robustness to model uncertainty, of the output gap, can be substantially increased by increasing either or both of the Taylor coefficients, it is now necessary to consider the dependence on g of the robustness of the interest rate increment, $\widehat{\alpha}_i(g, r_i, 1)$, shown in fig. 6. We see that the robustness of the interest rate decreases strongly as either

g_y or g_π increases. For example, at $g_\pi = 0.5$, the robustness decreases drastically as g_y increases: $\widehat{\alpha}_i = 2.28$ at $g_y = 0.1$, while $\widehat{\alpha}_i = 0$ at $g_y = 1.47$. Likewise, at fixed g_π, the peak robustness for any value of g_y decreases as g_π increases: $\max_{g_y} \widehat{\alpha}_i = 0.52$ at $g_\pi = 1.5$, while $\max_{g_y} \widehat{\alpha}_i = 0.21$ at $g_\pi = 4.5$.

In short, changes in the Taylor coefficients which improve the output-gap robustness (fig. 5), cause deterioration of the interest-rate robustness (fig. 6). The trade-off is very strong at larger values of g_y and g_π, with $\widehat{\alpha}_i$ reaching zero. In addition, the values suggested by Taylor ($g_\pi = 1.5$ and $g_y = 0.5$) are clearly infeasible, since $\widehat{\alpha}_i = 0$. There does not seem to be any satisfactory compromise, and in any case the robustnesses are everywhere rather small. The circle in fig. 6 has unit robustness: $\widehat{\alpha}_i(g_\pi = 0.5,\ g_y = 0.8) = 1$. However, the corresponding point in fig. 5 (also a circle) has lower immunity to model uncertainty: $\widehat{\alpha}_y(g_\pi = 0.5,\ g_y = 0.8) = 0.69$. As noted, a change in g which would increase $\widehat{\alpha}_y$ would also cause a decrease in $\widehat{\alpha}_i$. Finally, it is noted again that these numerical results depend on the specific inflation and output history, $\pi_{t(6)}$ and $y_{t(3)}$.

The fact of exceedingly low robustnesses has the following implication for Fed-watchers. If the US Fed does in fact use a Taylor rule to successfully regulate monetary policy, then either it is tremendously fortunate in the practice, or eqs.(10) and (11) do not in fact describe the US economy. Consequently, the supposition that the Fed uses a Taylor rule may be erroneous.

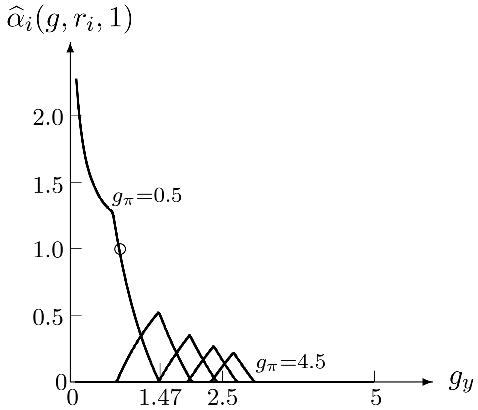

Figure 6: Robustness of interest rate increment vs. g_y, for $g_\pi = 0.5, 1.5, \ldots,\ 4.5$ (left to right). $\pi_{t(6)}$ and $y_{t(3)}$ same as in fig. 3. $r_i = 0.5$.

4 Methodological Summary

This paper develops a methodology for formulation and evaluation of monetary policy in anticipation of surprises which will strongly impact the economy. Two types of models are needed: models of dynamics and models of uncertainty. However, neither class of models can accurately predict surprises. Hence, especially regarding models of uncertainty, one should use exceedingly sparse and unassuming models. Attempts to identify worst cases, or likelihoods of extreme events, are unlikely to be accurate and can often lead the analyst astray. Furthermore, when policy is based on dynamic models, the basic **robustness question** which the analyst must address is: how wrong can the dynamic models be without jeopardizing adequate performance? We have referred to this approach as **robust satisficing** and the answer is embodied in the robustness function. This robustness question is in distinction to the question of optimization which is sometimes posed: what is the best performance which is attainable? We have shown that the attainment of optimal outcomes will always have zero immunity to modelling error.

The robustness function enables the analyst to compare policy options in terms of the level of model error up to which specified policy goals are still achieved. This is done without specifying the details of the surprises which underlie the modelling error. Rather, we quantify the modelling error as a fractional error of unknown size in the model coefficients. The robustness of a policy is the greatest fractional error at which goals are met.

The robustness function quantifies two aspects of the decision-making process: trade-off between robustness and performance, and reversal of preferences. The **trade-off** property states that the robustness to modelling-error decreases as the aspirations of the decision maker become more demanding. The robustness function quantifies how much performance must be sacrificed in order to obtain any specified level of immunity to modelling error. This trade-off is manifested in the monotonicity of the robustness functions as seen in figs. 1 and 3. One consequence of the trade-off property is that maximal aspirations — what the optimizing decision maker aims at — have zero robustness to surprises. This is illustrated by the robustness reaching zero in these figures.

The trade-off between robustness and performance may motivate the decision maker to relinquish performance-aspirations, and to "migrate up" the robustness curve to obtain positive robustness. **Reversal of preference** between policy options may occur when the robustness curves for the alternative policies cross, as in figs. 2 and 4. More robustness is preferred over less robustness, at the same level of performance. Consequently the decision maker who values robustness as well as performance may prefer an option which, at very low robustness, has poorer performance, and which at adequate (but sub-optimal) performance has higher robustness than the available alternatives.

In short, info-gap robust-satisficing is a methodology which enables policy selection with Pareto efficiency between performance and robustness, based on very limited information about possible modelling errors and surprises.

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