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Managing uncertainty through robust-satisficing monetary policy

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September 26, 2006

Abstract

We employ information-gap decision theory to derive a robust monetary policy response to Knightian parameter uncertainty. This approach provides a quantitative answer to the question: For a specified policy, how much can our models and data err or vary, without rendering the outcome of that policy unacceptable to a policymaker? For a given acceptable level of performance, the policymaker selects the policy that delivers acceptable performance under the greatest range of uncertainty. We show that such information-gap robustness is a proxy for probability of policy success. Hence, policies that are likely to succeed can be identified without knowing the probability distribution. We adopt this approach to investigate empirically the robust monetary policy response to a supply shock with an uncertain degree of persistence.

Keywords: *Knightian uncertainty, Monetary policy, Info-gap decision theory.*

JEL Codes: *E31, E52, E58, E61*

1 Introduction

In this paper, we employ 'information-gap decision theory' to derive robust monetary policy in the face of Knightian parameter uncertainty. An information-gap, hereafter info-gap, is

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a disparity between what *is* known, and what *must be* known, in order to make a reliable decision. An info-gap reflects a decision maker's ignorance about some aspect which cannot be modeled probabilistically. Thus, an info-gap is said to represent Knightian uncertainty about that aspect.

Info-gap decision theory deals with making robust decisions under Knightian uncertainty; see Ben-Haim (2006a). It suggests choosing a robust-satisficing policy in contrast with those implied by the two common approaches to decision making under uncertainty: the Bayesian approach and the robust decision theory; see e.g. Hansen and Sargent (2001), Hansen *et al* (2006) and Adam (2004). Within the info-gap approach, robustness to (specified) uncertainty is only available at the cost of sacrificing some performance relative to what is attainable through optimal monetary policy under no uncertainty. The more performance one is willing to sacrifice, the more robustness can be obtained.

Robustness of a policy may be equated with the range of possible realizations of the uncertain parameter within which acceptable or satisfactory outcomes are guaranteed by that policy. The larger the range, the higher the robustness (at a specified level of aspiration). Thus, the most robust policy would be the policy that delivers an acceptable outcome under the largest range of possible parameter values. Moreover, the lower the aspirations, i.e. the lower the required level of performance, the larger the range of parameter values under which the most robust policy will perform satisfactorily, i.e. not worse than the required level.

The main output from an info-gap-based analysis is a set of robustness functions associated with policies considered. A robustness function specifies how much robustness a given policy will deliver at different levels of required or acceptable performances. Accordingly, some policies may be more robust than others for all levels of acceptable performances, or they may be more robust than others up to some level of acceptable performance, while the converse may be the case afterwards.

The robust-satisficing policy would maximize robustness at a given level of acceptable performance. One may also say that the robust-satisficing policy would require a minimum of reduction in aspirations relative to the other policies considered to deliver a given robustness level.

We show that info-gap robustness may be considered a proxy for the probability of a policy's success. That is, any change in a policy which augments the robustness also augments the probability of a policy's success. Intuitively, an increase in the robustness of a policy can be equated with an increase in the number of parameter values under which the policy will

perform satisfactorily. Thus, if a policy becomes more robust, its success will tend to increase if the additional parameter value under which it would perform satisfactorily has a positive probability. Thus, even though the probability of a policy's success cannot be calculated under Knightian (parameter) uncertainty, the info-gap approach still enables us to rank policies consistent with their success probabilities and helps identify the policy with maximal success-probability.

In this paper, we employ the info-gap approach to derive robust-satisficing monetary policy in the face of a supply shock with an unknown degree of persistence. Specifically, we assume that uncertainty is confined to a single parameter, which determines the persistence in the shock to inflation in our model. The persistence parameter is assumed to vary in the range of $[0, 1)$ with unknown probability distribution. In order to identify the robust-satisficing policy among a set of policies in the face of the supply shock, we consider a set of monetary policy rules and derive robustness functions associated with each of them.

We proceed in three steps to derive the robust-satisficing policy. First, we derive values of the monetary policy objective function for all policies of interest under possible outcomes of the uncertain parameter in its prespecified space. Second, we evaluate how well each of the policies considered will perform under every possible outcome of the uncertain parameter. And third, we rank the policies in accordance with the number of possible parameter values, or the range of possible parameter values, under which they will perform not worse than various tolerable levels. The robust-satisficing policy would be the one that would deliver an acceptable outcome under the largest range of possible parameter values, i.e. has the highest degree of robustness.

The info-gap approach to deriving robust monetary policy under Knightian uncertainty has recently been discussed by Ben-Haim (2005b). The info-gap approach has, however, been previously applied to a wide variety of decision problems with Knightian uncertainty, including financial risk assessment (Ben-Haim 2005a), search behavior in animal foraging (Carmel and Ben-Haim 2005), policy decisions in marine reserve design (Halpern *et al* 2006), natural resource conservation decisions (Moilanen and Wintle 2006), inspection decisions by port authorities to detect terrorist weapons (Moffitt *et al* 2005a) and to detect invasive species (Moffitt *et al* 2005b), technological fault diagnosis (Pierce *et al* 2006) and engineering model-testing (Vinot *et al* 2005).

Studies of monetary policy decisions under uncertainty are mainly based on the Bayesian approach and the robust or maxmin decision theory; see e.g. Hansen and Sargent (2001), Onatski and Williams (2003), Levin and Williams (2003a), Levin *et al* (2003b), Leitemo and

Söderström (2005), Rustem *et al* (2005) and Tetlow and von zur Muehlen (2004). The Bayesian approach requires that one assigns a probability distribution on the uncertain aspect, e.g. model parameters. This enables one to choose an expected-loss-minimising policy. Under Knightian uncertainty, i.e. when uncertainty cannot be quantified in terms of (subjective) probabilities, one can proceed by assigning an equal probability to a set of possible outcomes. In such cases, the robust decision theory suggests policies that are designed to perform well in worst-case scenarios. Accordingly, a fictitious malevolent agent who represents a policy maker's worst fears concerning misspecification is introduced into the optimisation problem and motivates her to minimize the loss function under the worst-case scenario.¹

However, in contrast with the robust decision theory, the info-gap decision theory does not imply ameliorating the most extreme case, which the policy maker deems possible in practice. Instead, the info-gap approach guarantees a performance not worse than some required level under outcomes of the uncertain factor(s) within the largest possible range or space. In other words, our robust-satisficing strategy implied by the info-gap decision theory seeks a policy with adequate outcome for the largest range of unknown realizations.

Moreover, our robust-satisficing strategy does not require one to specify the worst case, which may be unknown. Actually, even when the worst case is known, ameliorating it is not necessarily the most robust response to Knightian uncertainty. For example, in our empirical analysis, the most robust policy corresponds to a scenario which is very different from the most extreme scenario. Another difference between the policy that ameliorates the worst case and the one that maximizes info-gap robustness may be probability of success. The info-gap strategy will maximize the probability of policy success, while a strategy that ameliorates the worst case may have low robustness and hence low probability of success. The satisficer relinquishes some aspiration and gains confidence - probability of success - in return.

In contrast with the Bayesian approach, the info-gap approach does not imply minimization of an expected loss function defined by a chosen set of probabilities over, say, possible parameter values. There is, however, one similarity with the Bayesian approach under a uniform probability distribution over possible parameter values. That is, initially one treats equally all possible parameter values within a prespecified range. But, instead of minimising expected loss (by assigning equal probability to all of the possible outcomes), one derives values of the

¹Knightian uncertainty can be either structured or non-structured (Tetlow and von zur Muehlen 2004). Under structured Knightian uncertainty, the true values of one or more specified parameters of the model are supposed to be bounded between known extreme values. Under unstructured Knightian uncertainty, however, few restrictions are placed on the uncertain aspects of a model. The case considered in this paper may therefore be classified as structured Knightian uncertainty.

objective function for all feasible policies conditional, in turn, on every value of the uncertain parameter in its prespecified range.

We employ a well documented macroeconometric model of the Norwegian economy to undertake the empirical analysis within the info-gap framework; see Bårdsen *et al* (2001) and Akram *et al* (2006) for model documentation. In order to ease the derivation of monetary policy rules in the light of the supply shock with some degree of persistence, and for expositional purposes, we follow the approach proposed in Akram (2006). Accordingly, a shock-specific linear interest rate rule is specified. This rule depends on a single decision variable: the policy horizon. A policy horizon is defined as the duration of the period in which the policy interest rate deviates from its reference value in response to a shock. The policy horizon is closely linked with the target horizon, which indicates when the inflation rate will be close to its target rate. Thus, one may distinguish between different (shock-specific) interest rate rules by different policy horizons, alternatively by different target horizons. Accordingly, the number of policies to be evaluated would be equal to the number of policy horizons of interest.

The paper is organized as follows. The next section offers a brief overview of the info-gap decision theory and reviews its main concepts. Section 3 formulates the decision problem within the context of the problem at hand, and shows that the degree of robustness can be considered a proxy variable for probability of policy success. The details of the proof are presented in Appendix 2, while Appendix 1 presents some regularity conditions that a loss function must satisfy to enable a precise formulation of the decision problem. Section 4 presents the loss function of the policy maker and the optimal monetary policy rule in the absence of parameter uncertainty. Section 5 characterizes the parameter uncertainty. Section 6 presents the empirical analysis. Section 7 contains the main conclusions.

2 Info-gap decision theory: Managing uncertainty

Info-gaps are non-probabilistic and cannot be insured against or modeled probabilistically. Hence, they are equated with Knightian uncertainty. Examples of common info-gaps include uncertainty regarding the shape of a probability distribution, the functional form of a relationship between entities, or the values of some key parameters.

Info-gaps are quantified by info-gap models of uncertainty. An info-gap model is an unbounded family of nested sets that share a common structure. A frequently encountered example is a family of nested ellipsoids that have the same shape. The structure of the sets in

an info-gap model depends on information about the uncertainty. In general terms, the structure of an info-gap model of uncertainty is chosen to define the smallest or strictest family of sets whose elements are consistent with the prior information.

A common example of an info-gap model is the fractional error model, which can be characterized as follows:

$$\mathcal{V}(\alpha, \tilde{v}) = \{v(x) : |v(x) - \tilde{v}(x)| \leq \alpha \tilde{v}(x)\}, \quad \alpha \geq 0. \quad (1)$$

Here, $\tilde{v}(x)$ denotes the best estimate of an uncertain function $v(x)$, while the fractional error from this estimate, α , is unknown. At any level of uncertainty α , the set $\mathcal{V}(\alpha, \tilde{v})$ contains all functions $v(x)$ whose fractional deviation from $\tilde{v}(x)$ is no greater than α . However, the level of uncertainty (α) is unknown, so the info-gap model is an unbounded family of sets, and there is no worst case or greatest deviation. Many other types of info-gap models are discussed in Ben-Haim (2006a).

All info-gap models obey two axioms: the axioms of nesting and contraction.

Nesting. The info-gap model $\mathcal{V}(\alpha, \tilde{v})$ is nested if:

$$\alpha < \alpha' \quad \text{implies} \quad \mathcal{V}(\alpha, \tilde{v}) \subset \mathcal{V}(\alpha', \tilde{v}). \quad (2)$$

Contraction. The info-gap model $\mathcal{V}(0, \tilde{v})$ is a singleton set containing its center point:

$$\mathcal{V}(0, \tilde{v}) = \{\tilde{v}\}. \quad (3)$$

The nesting axiom imposes the property of ‘clustering’, which is characteristic of info-gap uncertainty. Furthermore, the nesting axiom implies that the uncertainty sets $\mathcal{V}(\alpha, \tilde{v})$ become more inclusive as α grows, thus endowing α with its meaning as a level of uncertainty. The contraction axiom implies that at zero level of uncertainty the estimate \tilde{v} is true.

Uncertain variations may be either adverse or favorable. Adversity entails the possibility of failure. A robustness function expresses the greatest level of uncertainty at which failure cannot occur.²

More precisely, the robustness function can be expressed as the maximum value of the

²The case of favorable uncertainty is discussed in Ben-Haim (2006a).

uncertainty parameter α of an info-gap model:

$$\hat{\alpha}(q) = \max\{\alpha : \text{minimal requirements are always satisfied}\}. \quad (4)$$

Here, q denotes a vector of decision variables such as time of initiation, choice of a model or its parameters, or operational options. Equation (4) expresses that robustness of q , $\hat{\alpha}(q)$, is the greatest level of uncertainty α , or the greatest possible variation, for which specified minimal requirements are always satisfied. $\hat{\alpha}(q)$ expresses robustness — the degree of immunity against errors or deviations from ones' assumptions — so a large value of $\hat{\alpha}(q)$ is desirable.

The robustness function involves maximization of the uncertainty, or the range of variation in e.g. a variable, parameter or model, at which decision q would *satisfice* the performance at a tolerable level. This amounts to choosing actions or policies aimed at maximising robustness while satisficing performance. The robustness function specifies the trade-offs associated with a policy that one faces in a given situation.

More specifically, consider a scalar objective function or loss function $L(q, v)$, which depends on the decision vector q and the info-gap-uncertain function v . The minimal requirement in eq.(4) could be that the loss $L(q, v)$ be no greater than an acceptable level L^a . Usually L^a is not chosen irrevocably before performing the decision analysis. Rather, L^a enables the decision maker to explore a range of options (i.e. q -vectors).

The robustness function of eq.(4) can now be expressed more explicitly:

$$\hat{\alpha}(q, L^a) = \max \left\{ \alpha : \left(\max_{v \in \mathcal{V}(\alpha, \tilde{v})} L(q, v) \right) \leq L^a \right\}. \quad (5)$$

$\hat{\alpha}(q, L^a)$ is the greatest level of uncertainty or variation consistent with loss no greater than the tolerable loss L^a . This definition can be modified to handle multi-criterion loss functions.

The robustness function generates robust-satisficing preferences on the options. A decision maker will usually prefer q over q' if the robustness of q is greater than the robustness of q' at the same level of required performance L^a . That is:

$$q \succ_r q' \quad \text{if} \quad \hat{\alpha}(q, L^a) > \hat{\alpha}(q', L^a). \quad (6)$$

A robust-satisficing decision is one which maximizes the robustness on a set \mathcal{Q} of available

q -vectors and satisfies the performance at the acceptable level L^a :

$$\hat{q}_c(L^a) = \arg \max_{q \in \mathcal{Q}} \hat{\alpha}(q, L^a). \quad (7)$$

Usually, though not invariably, the robust-satisficing action $\hat{q}_c(L^a)$ depends on L^a .

A robustness function has the following key property. Robustness trades off against aspiration for outcome: robustness deteriorates as the decision maker's aspirations increase. In particular, robustness is zero when aspirations are at maximum. Accordingly, an optimizing decision conditional on specific values of relevant variables or parameters have zero robustness to alternative parameter values. Robustness curves of alternative decisions may cross, implying reversal of preference depending on aspiration. Moreover, in some cases, the probability of success is enhanced by enhancing the info-gap robustness, without knowing the underlying probability distribution. Thus, robustness is a proxy for probability of success. We will illustrate all of these properties in our analysis.

3 Robustness function under parameter uncertainty

In the following, we first adapt the relatively general robustness function eq.(4) to our case of Knightian parameter uncertainty. Thereafter, we present a theorem that relates our measure of robustness to the probability of policy success.

The uncertain parameter in our case is the degree of persistence in a supply shock denoted as ϕ . Sections 4 and 5 present our loss function, the value of which depends on ϕ , and the decision variable, H , which represents the policy horizon. We aim to control the relative loss denoted by $dL(H; \phi)$ and defined in Section 5.

3.1 Robustness function

Assume that we wish to satisfy the loss function $dL(H, \phi)$ at the value dL^a by deciding on an appropriate policy horizon, H , as we do in Section 5. That is, we require:

$$dL(H, \phi) \leq dL^a. \quad (8)$$

dL^a may be considered an exogenous value or a value which the policy maker chooses in the light of the associated robustness function. The value of persistence, ϕ , is unknown. The uncertainty in ϕ , α , may be represented by an info-gap model $\Phi(\alpha, H)$, which is specified in

Appendix 1.

The robustness of policy H , $\hat{\alpha}(H, dL^a)$, is the greatest level of uncertainty α up to which all realizations of the persistence result in acceptable loss to the decision maker:³

$$\hat{\alpha}(H, dL^a) = \max \left\{ \alpha : \left(\max_{\phi \in \Phi(\alpha, H)} dL(H, \phi) \right) \leq dL^a \right\}. \quad (9)$$

More robustness is better than less robustness, so we prefer policy H over H' if H has a higher robustness than H' , at the same value of dL^a :

$$H \succ_r H' \quad \text{if} \quad \hat{\alpha}(H, dL^a) > \hat{\alpha}(H', dL^a). \quad (10)$$

That is, the robustness function $\hat{\alpha}(H, dL^a)$ engenders a preference ranking on the policy options, H . However, this ranking need not be unique, since it depends on the policy goal expressed by the greatest acceptable loss, dL^a . Accordingly, the preferred policy induced by $\hat{\alpha}(H, dL^a)$ may change as the greatest tolerable loss, dL^a , changes.

The robust-satisficing policy, for acceptable loss dL^a , $\hat{H}(dL^a)$, is the policy which maximizes the uncertainty or range of variation in the persistence of the shock:

$$\hat{H}(dL^a) = \arg \max_H \hat{\alpha}(H, dL^a). \quad (11)$$

The robust-satisficing policy may differ from the full-knowledge optimal policy, which is defined in eq.(17).⁴

3.2 Robustness and probability of policy success

The robust-satisficing preference on the policy options, eq.(10), seems reasonable: a policy which is more immune to our ignorance of the persistence is desirable over a policy which is less immune. We can also relate the robustness of a policy to its probability of success. In the following, we show that robustness, which can be assessed, is a reliable proxy for the probability of success, which cannot be assessed without knowledge of the probability distribution of ϕ .

The policy, H , is successful if it entails a loss that is less than dL^a for the realized value of the persistence, ϕ , which is not known when the policy is chosen. Let $P(\phi)$ denote the

³Other examples of info-gap robustness functions for monetary policy can be found in Ben-Haim (2005b and 2006a, pp.75–78).

⁴In this paper we will not explore info-gap uncertainty in probability distributions. However, the robust-satisficing strategy may also differ from policies based on best-estimated probability distributions; for examples see Ben-Haim (2005a, 2006a).

unknown cumulative probability distribution for the persistence, with density function $p(\phi)$. Formally, the probability of success for policy H would be:

$$P_s(H, dL^a) = P[\phi : dL(H, \phi) \leq dL^a]. \quad (12)$$

We can now state the following theorem, which is proven in Appendix 2 and is related to other results in Ben-Haim (2006a, pp.279–284; 2006b). The proof depends on two concepts - coherence and weak convexity - which are defined in Appendix 2 building on Appendix 1.

Theorem 1 *The probability of a policy's success increases with its robustness.*

Given:

- *The info-gap model, $\Phi(\alpha, H)$ in eq.(26), is coherent with the probability distribution $P(\phi)$.*
- *The loss function, $dL(H, \phi)$, displays the weak convexity of eq.(23).*

Then:

$$\left(\frac{\partial \hat{\alpha}(H, dL^a)}{\partial H} \right) \left(\frac{\partial P_s(H, dL^a)}{\partial H} \right) > 0. \quad (13)$$

Theorem 1 asserts that any infinitesimal change in policy, H , which augments the info-gap robustness, $\hat{\alpha}(H, dL^a)$, also augments the probability of its success, $P_s(H, dL^a)$. The robustness function is calculated without knowledge of the probability distribution, so robustness may be considered a proxy for the probability of success. The theorem depends on the property of coherence, which imposes a weak constraint on the class of probability distributions. One can think of the coherence assumption as imposing an informational requirement on the info-gap model, which must reveal something about the probability distribution; see Appendix 2 for a precise definition of coherence.

4 Monetary policy: Objectives and instruments

In this section, we formulate the objective of monetary policy in the absence of Knightian parameter uncertainty.

To devise optimal response to an observable shock that occurs at time τ , the central bank minimizes the following loss function with respect to an interest rate path $i_\tau, i_{\tau+1}, i_{\tau+2}, \dots, i_{\tau+H-1}, i_{\tau+H}, i_{\tau+H+1}, \dots$:

$$L(.) = V_\tau(\Pi) + \lambda V_\tau(Y), \quad (14)$$

subject to a constraint that the inflation rate must become close to its target in the short or medium run rather than in the unspecified long run.⁵ H represents the policy horizon, i.e. the number of periods during which interest rates will deviate from their reference value and stimulate or restrain the economy. The target horizon, i.e. the number of periods inflation will deviate from its target, will generally be linked and be close to the policy horizon.

$V_\tau(\cdot)$ is a variance function conditional on information at time τ . λ indicates the degree of concern for real economic stability around a trend, Y relative to that of Π , which denotes deviation from the inflation target. The loss function admits a trade-off between the conditional variance of inflation and that of the output-gap.

The constraint regarding the time frame for achieving the inflation target can be embedded in an interest rate rule; see Akram (2006) for details. If the model is linear and interest rates are allowed to return gradually towards their reference value (i_0) upon a deviation, the following interest rate rule can serve this purpose:

$$i_t = i_0 + (1 - \varrho_H) \frac{\beta_\varepsilon}{(1 - \phi)} \varepsilon_\tau + \varrho_H (i_{t-1} - i_0) \quad ; \quad t = \tau, \tau + 1, \tau + 2, \dots \quad (15)$$

The response coefficient $(1 - \varrho_H) \beta_\varepsilon / (1 - \phi) \equiv \beta_{\varepsilon, H}$ determines how much the interest rate must change initially to counteract the inflationary effects of a shock ε_τ . This initial deviation is thereafter eliminated gradually, depending on the value of an interest rate smoothing parameter ϱ_H . It appears that both the response coefficient and the degree of smoothing depend on the policy horizon.⁶⁷

The value of $\beta_\varepsilon / (1 - \phi)$ depends on the shock and the model, and in this section is considered given. (We consider Knightian uncertainty in the next section.) Here, ϕ denotes the degree of persistence in the shock and is assumed to be positive and less than one. β_ε is a derived parameter whose value increases with the pass-through of the inflationary effects of

⁵This perspective, where achieving the inflation target can be considered the primary objective while obtaining a stable inflation rate and output can be considered secondary objectives, is consistent with what Faust and Henderson (2004) regard as best-practice monetary policy. Accordingly, "...best-practice policy can be summarized in terms of two goals: First get mean inflation right; second, get the variance of inflation right.", but "...getting the mean right may be the goal of greatest importance"; see Faust and Henderson (2004, pp. 117–118).

⁶This rule resembles a Taylor-type rule with interest rate smoothing except that it is the determinant of inflation, i.e. ε_τ , that enters the rule rather than inflation itself; see Taylor (1999) and the references therein.

⁷Alternatively, one could characterize monetary policy by a Taylor-type simple interest rate rule. Then, a specific policy would be defined by one possible combination of the response coefficients. Thus, a high number of policies would be possible, depending on the number of response coefficients, and had to be evaluated. In addition, the relationship between the uncertain parameter and the response coefficients would not be explicit. Another alternative could be to derive optimal interest rate rules, as in e.g. Estrella and Mishkin (1999). Such rules may be more suitable in the case of (theory-based) structural models. However, analytical expressions for such rules may be hard to derive in the case of econometric reduced-form models of some size.

the shock, but declines with the effectiveness of interest rates in checking inflation. It can be considered a constant (shock and model specific) parameter, if the transmission mechanism is super exogenous with respect to the policy changes considered; see Engle *et al* (1983). It follows that a persistent shock requires a stronger initial response than a transitory shock for a given degree of interest rate smoothing (ϱ_H).

The policy horizon enters the interest rate rule through the interest rate smoothing parameter. ϱ_H is defined as $\delta^{1/(H+1)}$ and takes on a value in the range of (0, 1) depending on the policy horizon H (for a given δ). δ is a sufficiently small fixed parameter of choice that indicates when the interest rate may be considered converged to its reference value. The degree of smoothing increases with the policy horizon in a concave fashion.

In particular, $H = 0$ will lead to (almost) no interest rate smoothing ($\varrho_H = \delta$) while large values of H will imply a high degree of interest rate smoothing since $\varrho_H = \delta^{1/(H+1)} \rightarrow 1$ when $H \rightarrow \infty$. The case $H = 0$ refers to the case when the policy maker only allows interest rates to deviate from their reference in a single period at time τ .

However, the initial response becomes stronger with a short policy horizon than with a relatively longer policy horizon. The value of the response coefficient $\beta_{\varepsilon,H}$ ($\equiv (1 - \varrho_H)\beta_{\varepsilon}/(1 - \phi)$) declines (in a geometric fashion) with the policy horizon or degree of interest rate smoothing. In particular, $(1 - \varrho_H)\beta_{\varepsilon}/(1 - \phi) \approx \beta_{\varepsilon}/(1 - \phi)$ when $H = 0$, while $(1 - \varrho_H)\beta_{\varepsilon}/(1 - \phi) \rightarrow 0$ when $H \rightarrow \infty$; since $\varrho_H \rightarrow 1$. This suggests that if a very long policy horizon is chosen, the interest rate needs to deviate only marginally from its reference value, but this deviation has to persist for a long time.

A long horizon would help subdue the required initial response to a relatively persistent shock. Especially, if persistence in a shock is matched by persistence in interest rates, i.e. $\varrho_H = \phi$, the initial response becomes equal to β_{ε} . In contrast, a short horizon may imply a particularly large deviation from the neutral level of interest rate in the face of a persistent shock. This may not even be feasible if the shock requires particularly low interest rates due to the zero bound on interest rates.

Clearly, the parameters characterizing the interest rate rule depend on the policy horizon (H), in a given model and a given i_0 . By varying H , one can vary the interest rate rule and thus the complete interest rate path as well as the level of the loss $L(\cdot)$. It follows that once the rule (eq.15) is implemented in the model, the optimal policy response to a shock can be found by minimizing the loss function (eq.14) with respect to the policy horizon H . The optimal value of H will then define the optimal value of β_{ε, H^*} , the optimal degree of smoothing ϱ_{H^*} ,

as well as the optimal level of loss.

We are particularly interested in analyzing the effect of persistence ϕ on the loss $L(\cdot)$ and consequently on the choice of policy horizon (H). It is therefore useful to express the loss function (eq.14) as an explicit function of H and ϕ :

$$L(\cdot) \equiv L(H, \phi). \quad (16)$$

The optimal horizon in the absence of Knightian parameter uncertainty can now be formally defined as:

$$H^*(\phi) = \arg \min_H L(H, \phi). \quad (17)$$

That is, if we had knowledge of ϕ , we could calculate the optimal policy, $H^*(\phi)$. The optimal policy horizon, H^* , will depend on the degree of concern for stability in the real economy (λ). Thus, β_{ε, H^*} and ϱ_{H^*} , will also depend on λ .

5 Monetary policy under Knightian parameter uncertainty

We now extend our analysis to include Knightian uncertainty in the persistence of shocks. We will derive a robust monetary policy response to a supply shock with unknown persistence. For simplicity, we also assume that one can infer the identity of the shock, its size and its stochastic process, though its duration is unknown. Specifically, we assume that the error term (ε) in the equation for domestic inflation in an empirical model can be characterized as an AR(1) process:

$$\varepsilon_t = \phi\varepsilon_{t-1} + v_t, \quad (18)$$

where ϕ is the degree of persistence that takes on an unknown value in the range of $[0, 1)$. v_t represents an unobservable IID-shock, but the supply shock ε_t can be observed.

The realized value of ϕ will generally affect the level of $L(H; \phi)$ and thereby the policy, *ceteris paribus*; cf. eq.(15). However, in order to evaluate the performance of a policy represented by H , its contribution to performance must be separated from that of the economic model and the realized value of ϕ .

One option is to evaluate feasible policies relative to the optimal policies under envisioned outcomes of the unknown ϕ .⁸ The difference in relative performance would imply the relative

⁸Another option could be to evaluate a policy relative to the optimal policy, conditional on one's best-guess

loss, $dL(H; \phi)$, by pursuing a policy that is different from whatever turns out to be the loss-minimizing policy. Thus, by controlling the relative loss $dL(H; \phi)$ rather than $L(H; \phi)$, one may control the loss due to the policy choice rather than to the ϕ -value. It is assumed that the policy choice does not influence the underlying process determining the ϕ -value.

The relative loss can be defined as:

$$dL(H; \phi) \equiv \frac{L(H; \phi) - L(H^*; \phi)}{L(H^*; \phi)}. \quad (19)$$

Here, $L(H; \phi)$ denotes the level of loss by choosing H conditional on a specific ϕ -value, while $L(H^*; \phi)$ expresses the loss under optimal policy given the specific ϕ -value. It follows that $dL(H; \phi) > 0$ for $H \neq H^*$ while $dL(H; \phi) = 0$ when $H = H^*$, when the loss function is continuous in H and there is a unique optimum.

For expositional purposes or for the purpose of mere ranking of policies, it is often useful to employ the following definition of the relative loss:

$$dL(H; \phi) \equiv \frac{L(H; \phi) - L(H^*; \phi)}{L(H; \phi)}. \quad (20)$$

This definition of the relative loss function allows one to confine the values of $dL(H; \phi)$ to the 0–1 range, which facilitates a graphical presentation of relative losses and their examination.

We proceed in three main steps to implement the approach for deriving robust monetary policy response to the supply shock.

First, we derive values of the loss function ($L(H; \phi)$) and relative losses $dL(H, \phi)$ for different monetary policy responses to a shock ε_t , conditional on every value of ϕ within the range of $[0, 1)$. We define the loss function by assuming λ equal to, say, 0.5, and let values of ϕ differ from each other by 0.1, for simplicity. A monetary policy response to a shock can be associated with a value of the policy horizon H . Accordingly, we derive values of the loss function for a wide range of policy horizons, say 0–12 quarters, representing monetary policy response to a shock with a specific ϕ . The optimal policy response (to a shock with a specific ϕ) can be represented by the optimal policy horizon, H^* in eq.(17), and the associated value of the loss function as $L(H^*, \phi)$.

We derive relative losses by pursuing different policies in response to a shock for each value value. How to form expectations about a parameter's value which is subject to Knightian uncertainty is not obvious, however. This is especially the case when the range of variation in a parameter or the uncertainty associated with it is unbounded. One possibility under Knightian uncertainty is, however, to form an expectation by placing an equal probability on all possible or plausible outcomes.

of ϕ (within its range). The relative loss $dL(\cdot)$ by pursuing some policy represented by policy horizon H is defined as in equation (20). The ranking of the policies would remain largely the same if we had used the definition (19).

Second, we derive losses by pursuing some policy H in response to the shock, conditional on different values of ϕ . Accordingly, to obtain values of $dL(H; \phi)$ for a specific policy, we vary ϕ within the range $[0-1)$, while keeping H fixed, and then repeat this procedure for a different H and so on.

And third, the information from the second step is used to find how much robustness different policies (H -values) offer at different levels of acceptable losses, dL^a . Robustness of a policy is measured by the range of ϕ for which the policy will imply lower loss than some acceptable loss, i.e. $dL(H; \phi) \leq dL^a$. A policy H' will have higher robustness than policy H'' if $dL(H'; \phi) \leq dL^a$ is true for a wider range of ϕ than $dL(H''; \phi) \leq dL^a$. Appendix 1 presents the info-gap model for uncertainty in the degree of persistence, upon which the robustness function is based, more precisely.

6 Empirical analysis

6.1 Model and monetary policy if persistence was known

We implement the above three steps in an econometric macromodel for Norway. The model used is a version of the model developed in Bårdsen *et al* (2003) that has been previously documented and used in Akram *et al* (2006). The model is (log) linear and estimated on quarterly aggregate data. It may be considered a backward-looking model in the sense that it does not make the expectation formation processes explicit. It is econometrically well-specified, however, with apparently invariant parameters with respect to changes in monetary policy over the sample: 1972–2001.

In the model, aggregate demand is determined by the real exchange rate, real interest rate and wealth effects from house prices and equity prices. Inflation is determined by a pro-cyclical markup on domestic production costs represented by unit labor costs and costs of imported inputs in domestic currency.⁹

We expose the model to a (supply) shock that raises inflation initially by one percentage

⁹The model also includes equations for credit demand, unemployment, wages, house prices, domestic equity prices and the nominal exchange rate. Monetary policy represented by a short run interest rate has direct effects on these asset prices, credit and aggregate demand in the short run, but is neutral in the long run; see Akram *et al* (2006) for details.

point, and thereafter allow for ten different degrees of persistence: 0, 0.1, 0.2,...0.9. For a supply shock the derived estimate of β_ε turns out to be 4; see Section 4 and Akram (2006) for more details. Estimates of the response coefficients $\beta_{\varepsilon,H}$ can be obtained from its formula: $(1 - \varrho_H)\beta_\varepsilon/(1 - \phi)$, for different degrees of persistence in the shock and interest rates, ϕ and ϱ_H , respectively. Values of ϱ_H for different policy horizons are obtained from $\varrho_H = \delta^{1/(H+1)}$, where δ is set to 0.1. This implies that we would consider an interest rate approximately equal to its reference value if it deviates not more than 1/10 of a percentage point from the reference value. Alternative values of δ do not bring about substantially different results.

The left frame of Figure 1 displays values of the response coefficient $\beta_{\varepsilon,H}$ for various degrees of persistence, ϕ , vs. the policy horizon, H , in the range from 0–12 quarters. It is evident that an increase in persistence raises the required interest rate response upwards for all policy horizons. For example, if $H = 0$, an increase in the inflation rate by one percentage point due to a transitory shock ($\phi = 0$) would require an increase in interest rate by 4 pp relative to its reference value. However, if the shock is persistent, the required increase would be 5 pp, 6.7 pp, and 10 pp, for degrees of persistence equal to 0.2, 0.4 and 0.6, respectively. When $H = 0$, however, interest rates would be brought back almost immediately, as $\varrho_H = \delta = 0.1$.

Figure 1 also illustrates that an increase in the policy horizon contributes to reduce the required initial interest rate response $\beta_{\varepsilon,H}$ (left frame), but raises the degree of interest rate smoothing ϱ_H (right frame). For example, the required initial interest rate responses, $\beta_{\varepsilon,H}$, when $\phi = 0.4$ and $\phi = 0.6$ declines from 6.7 and 10 to 1.4 and 2.1, respectively, if the policy horizon is 10 quarters rather than zero. This must, however, be accompanied by an increase in interest rate smoothing, ϱ_H , from 0.1 to 0.81 (right frame).

Figure 2 presents values of relative loss functions, $dL(H; \phi)$ in response to the supply shock under different horizon-specific interest rate rules. The figure shows values of $dL(H; \phi)$ under thirteen different policies, represented by policy horizons in the range of 0–12 quarters, for each value of ϕ from 0–0.9 with step size 0.1. The values of loss functions $L(H, \phi)$ and $dL(H; \phi)$ have been obtained by implementing interest rate rules, adapted to the different degrees of persistence in the supply shock, for different policy horizons and simulating the model over a period of six years: 1995q1–2000q4. Thereafter, the conditional variances of inflation and output have been calculated using the obtained results from the simulations. Values of the loss function $L(H; \phi)$, which is defined by eq.(14), have been obtained by combining the variances assuming $\lambda = 0.5$, while those of the relative loss function $dL(H; \phi)$ have been obtained by employing the definition eq.(20).

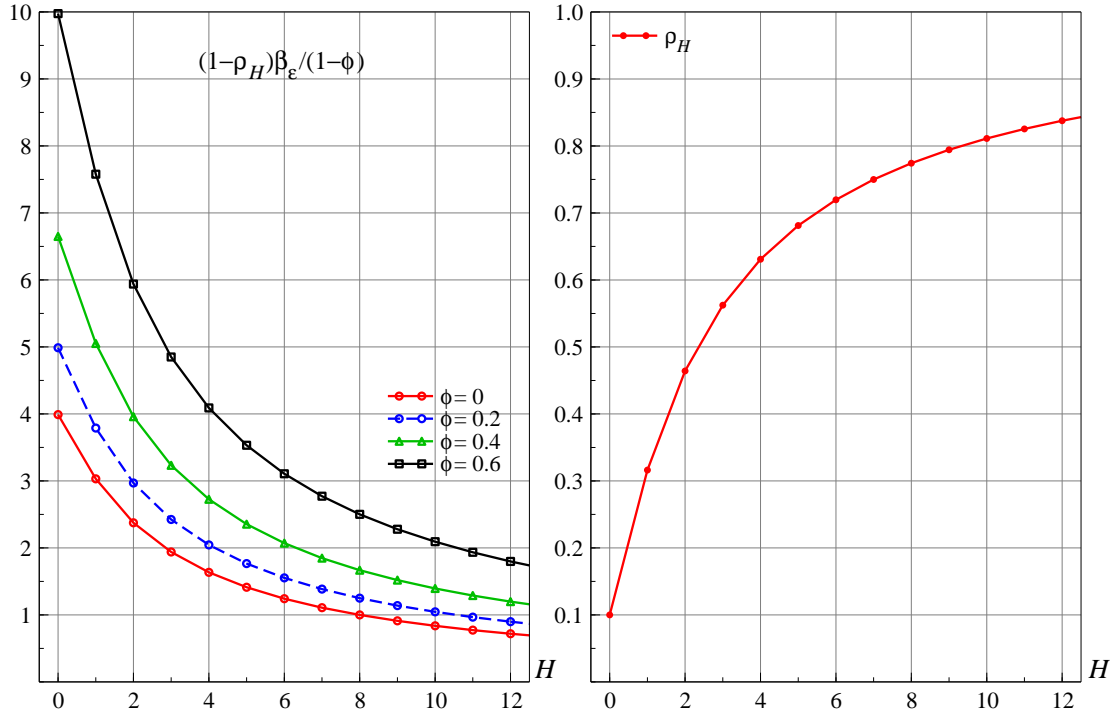


Figure 1: *Left: Initial interest rate response, $\beta_{\varepsilon,H}$, to supply shocks with different degrees of persistence, ϕ . The initial interest rate response is implied by policy horizons in the range 0–12 quarters. Right: Interest rate smoothing, ρ_H , associated with different policy horizons.*

It appears that the relative loss $dL(H; \phi)$ has a unique minimum value. Values of $dL(H; \phi) = 0$ indicate that the associated value of H would be the optimal policy (alternatively, the loss minimising policy) for the given degree of persistence ϕ , while values of $dL(H; \phi) > 0$ indicate that the associated policy (H) would be suboptimal. In the case of $\phi = 0.3$, it seems that $dL(H; \phi) = 0$ for two different but adjacent values of H . This can be explained as an artefact of the model where a change in H by one quarter implies only a negligible change in the loss function.

Figure 2 suggests that the optimal horizon $H^*(\phi)$, conditional on specific degrees of persistence (ϕ -values), increases with the degree of persistence. A long horizon amounts to choosing a higher degree of interest rate smoothing and lower initial response; see Figure 1. Thus, one may say that persistence in a shock favors persistence in interest rates as well. There are a few exceptions from this pattern which may be explained by the fact that an increase in the degree of persistence, of say 0.1, is too small to imply a shift in the policy horizon by a whole quarter. A model at a higher frequency, e.g. monthly, may have provided a more continuous relationship between ϕ and $H^*(\phi)$.

The intuition for the positive relationship between the degree of persistence and optimal

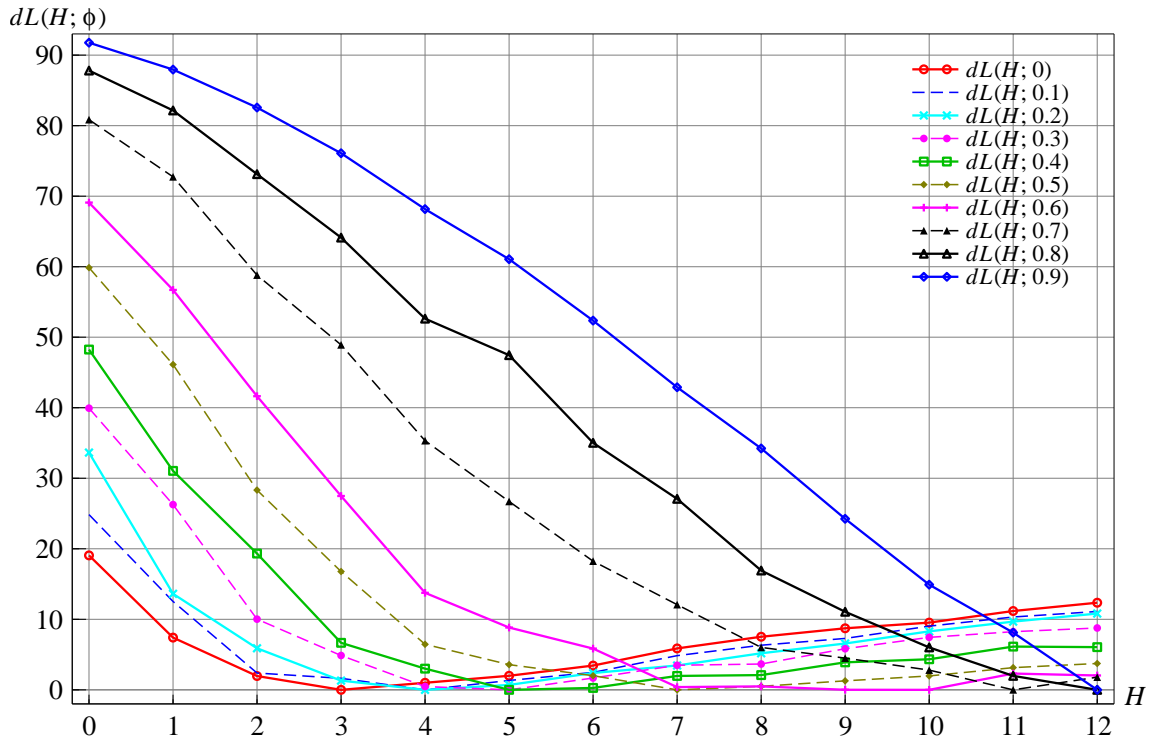


Figure 2: Values of $dL(H; \phi)$, which is defined by eq.20, on the vertical axis vs. the policy horizon in number of quarters on the horizontal axis, for various degrees of persistence, ϕ .

horizon is that the bulk of the effects of persistent shocks emerge farther into the future than those of relatively transitory shocks. Thus, if a short horizon is chosen, it would require a much stronger immediate response whose main effects would emerge earlier than those of persistent shocks. This would contribute to higher variances of inflation and output and hence higher loss. In contrast, if a longer horizon is chosen, the stabilizing effects of monetary policy would emerge more synchronous in time with those of persistent shocks. Hence, monetary policy would prove to be relatively more stabilizing. The optimal horizon minimizes the loss function by synchronizing the effects of a shock with those of monetary policy as much as possible.

Figure 2 displays largely a convex relationship between the loss function $dL(H, \phi)$ and the degree of persistence, as assumed in Appendix 1, eq.(23). Accordingly, it becomes more and more costly as the true persistence deviates from the assumed persistence, which justifies some optimal horizon H^* . Furthermore, the loss is asymmetrically distributed around the optimal policies (H^* 's). It can be more costly to choose a shorter horizon than to choose a longer horizon.

The asymmetric shape of the relative loss functions depends largely on the form of the monetary policy rule, as the model can be considered linear. Specifically, the monetary policy

rule embeds a concave relationship between the degree of interest rate smoothing and the policy horizon; see Figure 1. Thus, an increase in the policy horizon from a low level has a larger effect on the interest rate rule and consequently on the loss function than an increase in the policy horizon from a relatively high level.

A linear relationship between the degree of interest rate smoothing and the policy horizon would have contributed to a symmetric distribution of the (relative) losses around their optimal levels. The optimal horizons would be affected if the linear relationship is implemented. However, the results presented and to follow do not change qualitatively. Hence, our conclusions remain the same in general terms.¹⁰

6.2 Robust-satisficing monetary policy

Figure 3 presents robustness curves of different policies at different levels of acceptable losses, measured relative to the optimum levels. The vertical axis shows the fraction of the range for ϕ values for which a specific policy represented by the policy horizon will deliver a lower loss than the acceptable loss: $dL(H; \phi) \leq dL^a$. Different levels of acceptable losses (dL^a) are measured on the horizontal axis in percent of deviations from optimum levels. The fraction 1 corresponds to the full range: 0-0.9, which constitutes 10 possible values. Thus, values of e.g. 0.5 and 0.7 indicate robustness against 5 and 7 out of the 10 possible ϕ -values, respectively. The values constituting the different ranges can be ascertained from Figure 2. Accordingly, the fractions 0.5 and 0.7 refer to the ranges 0-0.4 and 0-0.6, respectively, when $dL^a = 10\%$. In the special case of fraction 0.1, the subrange of possible ϕ -values consists of a single ϕ value within the range 0-0.9, and can be ascertained from Figure 2.

We observe a positive relationship between the levels of acceptable loss, dL^a , and the degree of robustness, $\hat{\alpha}(H, dL^a)$. That is, there is a trade-off between robustness and desired closeness to the optimal level under a given outcome of ϕ . Hence, robustness comes at a price in terms of deviation from the performance under the optimal policy.

Figure 3 shows that some policies are generally more robust than others. They have a higher robustness than others at all levels of acceptable loss. In particular, policies associated with long policy horizons tend to be more robust than those with relatively shorter horizons. Accordingly, a given level of robustness can be achieved by accepting lower deviations from

¹⁰We obtained a linear version of the rule eq.(15) by assuming $\varrho_H = \mu H$, where μ was set to some numerical value, e.g. 0.08, such that $\varrho_H < 1$ for all values of H considered. Accordingly, $\beta_{\varepsilon, H}$ ($\equiv (1 - \varrho_H)\beta_{\varepsilon}/(1 - \phi)$) also becomes a linear function of H ; cf. Section 4. A presentation of the numerical results in the case of the linear rule has been left out, but is available upon request.

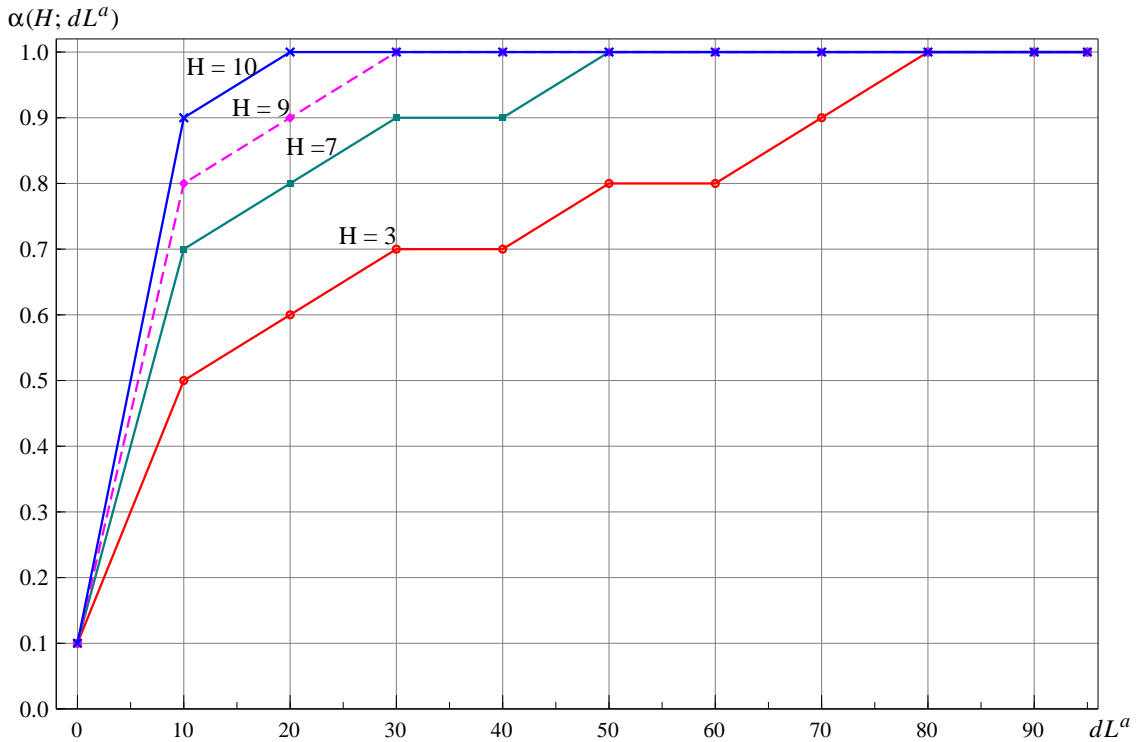


Figure 3: *Robustness functions for monetary policies associated with different policy horizons (H). The robustness on the vertical axis indicates the share of parameter range under which a given policy represented by the policy horizon will deliver a relative loss ($dL(H; \phi)$) not greater than that indicated on the horizontal axis.*

the optimum level with long policy horizons than with short policy horizons.

For example, Figure 3 suggests that the policy horizon of 10 quarters is able to provide robustness against 9 outcomes of ϕ values up to 0.8 if one sets the acceptable loss at 10%. In comparison, a policy horizon of 3 quarters would require an acceptable loss at 70% to deliver the same level of robustness. For acceptable loss at 10%, the policy horizon of 3 quarters would deliver a loss up to 10% for just 5 out of 10 ϕ values in the range 0-0.4. Notably, if one sets acceptable loss at 20%, the policy represented by 10 quarters will enable one to achieve full robustness, denoted by 1. That is, this policy will never deliver a relative loss higher than 20% under any of the ten possible ϕ values in the range 0-0.9. In contrast, the policy horizon of 3 quarters requires willingness to accept relative losses up to 80% in order to obtain full robustness.

The policy horizon of 10 quarters would represent optimal policy if persistence turns out to be 0.6, i.e. $H * (\phi = 0.6) = 10$. With a policy horizon of $H=10$, the response to a supply shock that initially raises inflation by 1 percentage point would be to raise the interest rate by about 2 percentage points relative to its reference value and thereafter bring it back gradually over

10 quarters. This implies an interest rate smoothing equal to 0.81; see Figure 1. Interestingly, this value is close to that reported in several empirical studies; see e.g. Sack and Wieland (2000).

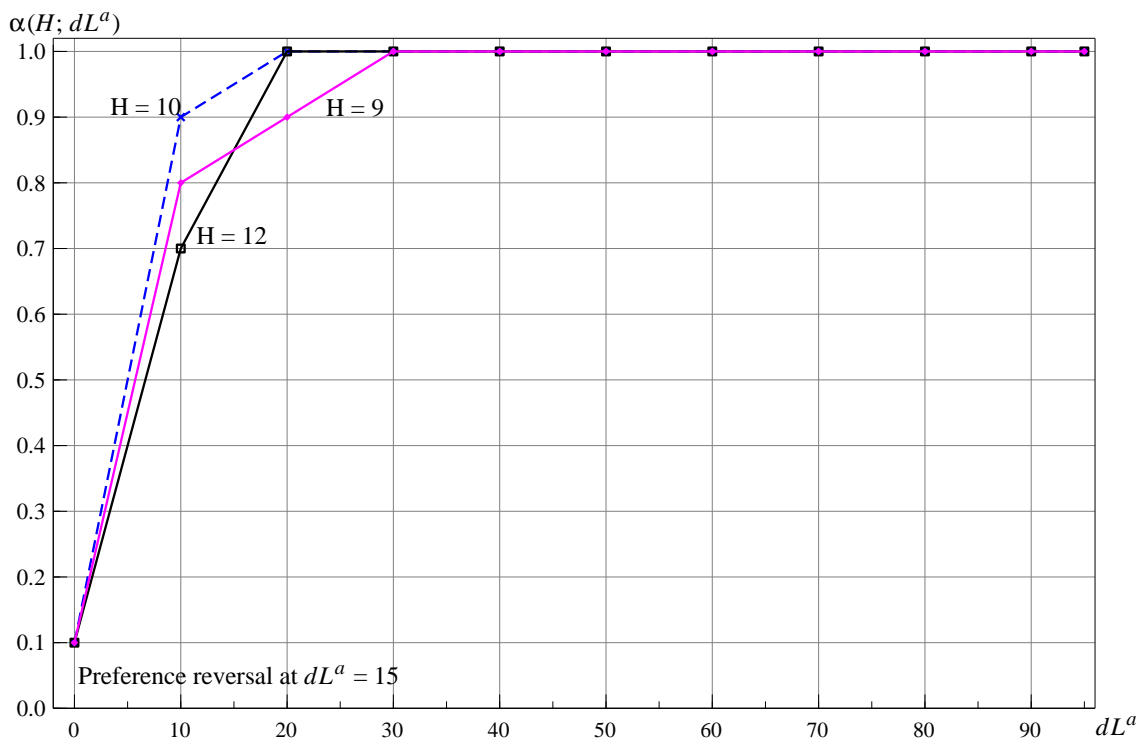


Figure 4: *Robustness-functions for monetary policies associated with different policy horizons (H). The figure illustrates the case of preference reversal when $dL^a = 15\%$.*

Figure 4 demonstrates the case of preference reversals regarding robustness of policies. We note that a policy horizon of 9 quarters dominates the policy with policy horizon of 12 quarters at relatively low levels of acceptable losses, while the opposite is the case at relatively higher levels of acceptable losses. More precisely, a policy horizon of 9 quarter is preferred over a policy horizon of 12 quarters for $dL^a < 15$, while the converse is the case for $dL^a > 15$. The figure also shows that the policy horizon of 10 quarters dominates the policy horizons of 12 and 9 for $dL^a < 20$ and $dL^a < 30$, respectively. For $dL^a = 20$ and $dL^a = 30$, the policy horizon of 10 quarters provide the same level of robustness as the policy horizons of 12 and 9 quarters, respectively. However, the policy horizon of 10 quarters would be preferred, as the possible loss incurred would never exceed that implied by the alternative policies horizons, irrespective of the realized ϕ -value.

To summarize, the above analysis suggests that relatively long policy horizons tend to be associated with more robust policy than short policy horizons. This implies that a less

aggressive but persistent monetary policy stance will deliver a more acceptable performance up to a higher degree of shock persistence than an aggressive policy stance. This supports gradualism in interest rate setting to a large extent.

In the case of the supply shock considered, monetary policy associated with a policy horizon of 10 quarters dominates all other policies associated with shorter or longer horizons. This defines the optimal policy if persistence is actually 0.6. This demonstrates that the robust policy differs from the optimal policy under the worst-case scenario, which may be considered the case when the degree of persistence is 0.9, due to its relatively large effect on the loss function.

The robust policy would be suboptimal if the degree of persistence turns out to be different from 0.6; see Figure 2. This is clearly the case if the supply shocks turn out to be transitory with no persistence or highly persistent. However, one can be confident that the loss, relative to whatever turns out to be the optimal level, does not exceed 10% for $\phi \in [0, 0.8)$ and 20% for $\phi \in [0, 0.9)$.

7 Concluding remarks

We have adapted the information-gap decision theory to derive robust monetary policy when there is Knightian parameter uncertainty. This approach allows one to manage uncertainty by choosing a policy that delivers an acceptable performance for a known range of parameter outcomes. Robust-satisficing policies maximize that range for tolerable levels of performance. Furthermore, it is proved that no other policy has greater probability of success.

We have illustrated this approach by deriving a monetary policy response to a supply shock whose degree of persistence is uncertain in a Knightian way. We have used an econometric model for Norway to undertake the empirical analysis. For convenience, we have characterized monetary policy by rules that can be fully specified by a single decision variable: the policy horizon. The results suggest that relatively long policy horizons or interest rate rules with a relatively high smoothing parameter tend to be more robust than those with short policy horizons, or a low degree of smoothing.

Several cases in our empirical analysis demonstrate that robustness is achieved in exchange for suboptimal performance. These costs are brought about by uncertainty — a gap in our information set — namely, our ignorance regarding the true degree of persistence. The approach outlined in this paper suggests a way to manage this uncertainty by choosing a policy

that limits the possible deterioration in performance to some acceptable level irrespective of how persistent the shock turns out to be.

Of course, the robustness of our results needs to be examined by considering alternative classes of interest rate rules and/or models. The approach outlined can be adapted easily to such tasks and to deriving robust-satisficing policy when facing other sources of uncertainty including model and data uncertainty.

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Appendix 1: Info-gap model of uncertainty in persistence

Here, we formulate the info-gap model, $\Phi(\alpha, H)$, for uncertainty in the persistence, ϕ , for which the analyst has complete Knightian uncertainty: the probability distribution of ϕ is unknown. One has no estimate, in any statistical sense, of the value of ϕ .¹¹

Even though the true value of ϕ is not known, it is still useful to talk about values of ϕ that would motivate any particular choice of policy horizon H . For instance, if we are considering a specific value of H , one may ask: given our economic models, what should ϕ be in order to make this a good choice of H ? There is nothing un-Knightian (or un-info-gap) about this question. We can answer this question without supposing anything about the probability distribution of ϕ . The value of ϕ which, were it the true value, would justify a particular H , will be denoted $\tilde{\phi}(H)$ and is defined implicitly in the relation:

$$H = H^*(\phi), \tag{21}$$

where $H^*(\phi)$ is the full-knowledge loss-minimizing policy if the persistence equals ϕ . The solution of this relation, for ϕ , may not be unique, in which case we define $\tilde{\phi}(H)$ as the lowest such solution:

$$\tilde{\phi}(H) = \min \{ \phi : H = H^*(\phi) \}. \tag{22}$$

We now posit a “weak convexity” of the loss function. We assume that, for fixed H :

$$\frac{\partial [dL(H, \phi)]}{\partial |\phi - \tilde{\phi}(H)|} > 0. \tag{23}$$

That is, for a fixed H , the loss function $dL(H, \phi)$ rises increasingly above the full-knowledge value, $dL[H, \tilde{\phi}(H)] = 0$, as ϕ deviates from $\tilde{\phi}(H)$. Relation (23) does not assert that $dL(H, \phi)$ is convex vs. ϕ , but only that it has a unique minimum vs. ϕ in $[0, 1)$, for any given H . We posit that this weak convexity holds for $dL(H, \phi)$. (Figure 2 illustrates that this is indeed the case for the macromodel used in our example.)

The weak convexity property implies that, for any non-negative bound on the loss function, the corresponding set of ϕ -values is a simple interval, which we define as:

$$\mathcal{D}(x, H) = \{ \phi \in [0, 1) : dL(H, \phi) \leq x \}. \tag{24}$$

¹¹This example could be extended to consider info-gap uncertainty in the probability distribution of ϕ . Examples of info-gap analysis of uncertainty probability distributions are found in Ben-Haim (2005a, 2006a).

For any level of loss x , the set of ϕ values for which the loss (with policy H) does not exceed x is the set $\mathcal{D}(x, H)$. Because the loss function has the property of weak convexity, this set is an interval whose length we denote by $|\mathcal{D}(x, H)|$. Since $dL(H, \phi) = 0$ at its unique minimum, $\tilde{\phi}(H)$, we note that $\mathcal{D}(0, H) = \{\tilde{\phi}(H)\}$.

The weak convexity property implies that the sets $\mathcal{D}(x, H)$ are nested:

$$x < x' \quad \text{implies} \quad \mathcal{D}(x, H) \subseteq \mathcal{D}(x', H). \quad (25)$$

This states that any persistence, ϕ , whose loss is no less than x , also has loss no less than x' , when the same policy is used.

We use this concept to define an info-gap model for uncertainty in ϕ . For any contemplated policy horizon H , the info-gap model is the following family of nested sets of persistence values:

$$\Phi(\alpha, H) = \{\phi : \phi \in \mathcal{D}(x, H), x \geq 0, |\mathcal{D}(x, H)| \leq \alpha\}, \quad \alpha \geq 0. \quad (26)$$

At any level of uncertainty α , the set $\Phi(\alpha, H)$ contains all persistence values ϕ which belong to sets $\mathcal{D}(x, H)$ no larger than α , regardless of the loss value x . The level of uncertainty is unknown so α can take any non-negative value. The info-gap model is a family of nested sets and obeys the axioms of contraction and nesting discussed in Section 2 (Ben-Haim 2006a).

In light of eq.(25) we see that, for any α , there exists an $x(\alpha)$ such that:

$$\Phi(\alpha, H) = \mathcal{D}(x(\alpha), H). \quad (27)$$

Combining eqs.(24) and (27) we see that the uncertainty set, evaluated at a level of uncertainty equal to the robustness, is:

$$\Phi[\hat{\alpha}(H, dL^a), H] = \mathcal{D}(dL^a, H). \quad (28)$$

We emphasize that this info-gap model depends on a contemplated policy horizon H . The sets $\Phi(\alpha, H)$ are an expression of epistemic (rather than objective or ontological or aleatoric) uncertainty: if we use policy H , then the info-gap model contains all persistence values for which the loss will not exceed some specific value. We don't know which ϕ value will occur or the value of α , so there is no known worst case (other than unbounded persistence).

Appendix 2: Proof of Theorem 1

Let $P(\phi)$ denote the unknown cumulative probability distribution for the persistence, with density function $p(\phi)$. From the definition of $\mathcal{D}(x, H)$ in eq.(24) we see that the set of all persistence values for which policy H is successful is $\mathcal{D}(dL^a, H)$. Thus, formally, the probability of success for policy H is:

$$P_s(H, dL^a) = P[\mathcal{D}(dL^a, H)]. \quad (29)$$

which is precisely eq.(12).

From the property of weak convexity we know that $\mathcal{D}(dL^a, H)$ is a simple interval, which we denote:

$$\mathcal{D}(dL^a, H) = [\phi_1(H), \phi_2(H)], \quad (30)$$

where the end-point functions $\phi_i(H)$ are known.¹²

We now define the property of *coherence* between the info-gap model, $\Phi(\alpha, H)$ in eq.(26), and the probability distribution $P(\phi)$. For notational convenience we define $p(\phi_i) = p[\phi_i(H)]$ and $\phi'_i = d\phi_i(H)/dH$.

Definition 1 *The info-gap model, $\Phi(\alpha, H)$ in eq.(26), and the probability distribution $P(\phi)$, are coherent at H if:*

$$(\phi'_2 - \phi'_1)[p(\phi_2)\phi'_2 - p(\phi_1)\phi'_1] > 0. \quad (31)$$

We can understand the meaning of coherence with the help of eqs.(28) and (30). A change in the policy horizon, H , causes the info-gap model to shift, through movement of the endpoints $\phi_i(H)$ of $\mathcal{D}(dL^a, H)$. Policy change also alters the corresponding probability densities, $p(\phi_i)$. The info-gap model and the probability distribution are coherent, in the sense of eq.(31), if shifts in $p(\phi_i)$ are not disproportionately different from the shifts in ϕ_i .

Proof of theorem 1. From relation (28) and the definition of $\Phi(\alpha, H)$ in eq.(26) we see that:

$$\hat{\alpha}(H, dL^a) = |\mathcal{D}(dL^a, H)|. \quad (32)$$

From the property of weak convexity we know that $\mathcal{D}(dL^a, H)$ is an interval whose length is $\phi_2(H) - \phi_1(H)$. Consequently, changes in robustness, $\hat{\alpha}(H, dL^a)$, resulting from changes in

¹²Whether the interval in eq.(30) is open or closed is immaterial for our argument; we assume closedness for notational simplicity.

policy, H , arise from alteration of the endpoints of $\mathcal{D}(dL^a, H)$. Specifically, from eq.(32):

$$\frac{\partial \hat{\alpha}(H, dL^a)}{\partial H} > 0 \quad \text{if and only if} \quad \phi'_2 - \phi'_1 > 0. \quad (33)$$

Expression (29) relates the probability of policy success, $P_s(H, dL^a)$, to the persistence interval $\mathcal{D}(dL^a, H)$. This relation shows how the probability of success varies with changes in policy. Using the succinct notation introduced after eq.(30), we find:

$$\frac{\partial P_s(H, dL^a)}{\partial H} = p(\phi_2)\phi'_2 - p(\phi_1)\phi'_1. \quad (34)$$

Thus:

$$\frac{\partial P_s(H, dL^a)}{\partial H} > 0 \quad \text{if and only if} \quad p(\phi_2)\phi'_2 - p(\phi_1)\phi'_1 > 0. \quad (35)$$

So, from the assumed coherence of the info-gap model with the probability distribution, and combining eqs.(33) and (35), we obtain eq.(13) which completes the proof.

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