Choose a Nature Reserve

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The problem. We must select between several alternative nature reserves. We have estimated the utility (e.g., duration until biodiversity will be threatened) of these alternatives. However, these estimates are highly uncertain. Nonetheless, a choice must be made. We will illustrate the info-gap robust-satisficing and opportune-windfalling strategies.

Formulation. For each candidate reserve we know a low- and high-utility estimate, where the probability that the reserve will have the low-utility value is p, and the probability that the reserve will have the high-utility value is $1-p$. We know the value of p confidently, but the values of low- and high-utility are uncertain. Our estimates of the low- and high-utility for the ith reserve are $\widetilde{u}_{i0}$ and $\widetilde{u}_{i1}$. Furthermore, we have error-estimates for these values, which we denote σ_{i0} and σ_{i1}.

Uncertainty, satisficing and windfalling. The low- and high-utilities of each nature reserve are highly uncertain, as represented in this fractional-error info-gap model for the ith reserve:

$$\mathcal{U}_i(h) = \left\{ u : \left| \frac{u_{ij} - \widetilde{u}_{ij}}{\sigma_{ij}} \right| \le h, \ j = 0, \ 1 \right\}, \quad h \ge 0 \tag{1}$$

The best estimate of the expected utility of the ith reserve is $\mathrm{EU}_i(\widetilde{u}) = p\widetilde{u}_{i0} + (1-p)\widetilde{u}_{i1}$. The actual value of the expected utility, $\mathrm{EU}_i(u_i)$, is unknown, since the utility-vector u_i is unknown. We require that this utility be no worse than a critical value, E_{c}:

$$\mathrm{EU}_i(u_i) \ge E_{\mathrm{c}} \tag{2}$$

This is a critical requirement which it is very important to obtain. Eq.(2) is a **satisficing** requirement.

A windfall aspiration is that the expected utility be as large as E_{w}, where E_{w} is greater than the estimated utility. The windfall aspiration is:

$$\mathrm{EU}_i(u_i) \ge E_{\mathrm{w}} \tag{3}$$

We do not require the attainment of expected utility this large, though if it happened this would be wonderful. Eq.(3) is a **windfalling** aspiration.

Robustness and opportuneness. The **robustness** to uncertainty of the ith nature reserve is the greatest horizon of uncertainty up to which the expected utility of that reserve is guaranteed to satisfy the critical requirement, eq.(2):

$$\widehat{h}(i) = \max \left\{ h : \left(\min_{u \in \mathcal{U}_i(h)} \mathrm{EU}_i(u) \right) \ge E_{\mathrm{c}} \right\} \tag{4}$$

The **opportuneness** from uncertainty of the ith nature reserve is the lowest horizon of uncertainty at which the expected utility of that reserve can (but does not necessarily) satisfy the windfall aspiration, eq.(3):

$$\widehat{\beta}(i) = \min \left\{ h : \left(\max_{u \in \mathcal{U}_i(h)} \mathrm{EU}_i(u) \right) \ge E_{\mathrm{w}} \right\} \tag{5}$$

Example. We now evaluate the robustness and opportuneness functions for 3 candidate nature reserves. The available information is:

$$[\widetilde{u}_1 \ \widetilde{u}_2 \ \widetilde{u}_3] = \begin{bmatrix} 20 & 22 & 18 \\ 25 & 27 & 21 \end{bmatrix}, \quad [\sigma_1 \ \sigma_2 \ \sigma_3] = \begin{bmatrix} 5 & 7 & 4 \\ 6 & 9 & 6 \end{bmatrix}, \quad p = 0.3 \tag{6}$$

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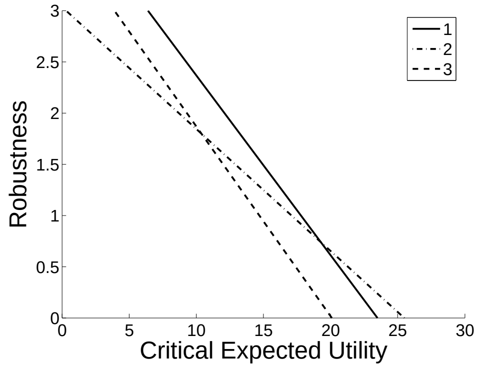

Figure 1: Robustness curves for 3 nature reserves.

Fig. 1 shows robustness curves for the three nature reserves specified in eq.(6). Each curve hits the horizontal axis at the estimated value of expected utility for that reserve.

Reserve 2 (dot-dash) has the highest estimated expected utility. However, the robustness is zero for E_c-values on the axis. Reserve 2 has the lowest slope which means that it obtains substantial robustness only by giving up substantial expected utility.

Reserve 1 (solid) has lower estimated expected utility than reserve 2, but reserve 1 has a steeper curve, meaning that robustness is less expensive in units of expected utility for reserve 1 than for reserve 2.

Reserve 3 (dash) is robust-dominated by reserve 1 over the range of E_c values shown.

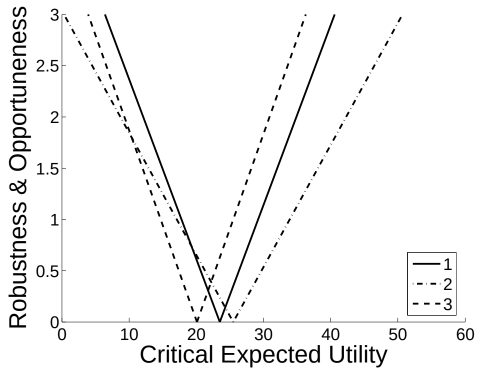

Figure 2: Robustness and opportuneness curves for 3 nature reserves.

Examples of robustness and opportuneness curves are shown in fig. 2 for the three nature reserves specified in eq.(6). The robustness curves (negative slopes) are reproduced from fig. 1.

Recall that a small value of the opportuneness function, $\widehat{\beta}$, is desirable, since small $\widehat{\beta}$ means that windfall is possible at very low uncertainty.

The opportuneness curves have positive slope, expressing the trade-off between large windfall (large E_w) and small ambient uncertainty (small $\widehat{\beta}$).

We note that the opportuneness curves of the 3 reserves do not cross one another. One can show that if the robustness curves for reserves i and j *do* cross one another, then their opportuneness curves *do not* cross. The significance of this for choosing a nature reserve is that, when the robustnesses are equal, the opportunenesses can be used to break the tie.

For instance, at critical utility of 20, we see that reserves 1 and 2 (solid and dot-dash) have the same robustness (about $\widehat{h} = 0.7$) since the curves cross one another. However, reserve 2 is consistently more opportune than reserve 1, so if $E_c = 20$ is accetable, and if the corresponding robustness seems adequate, then one might be inclined to prefer reserve 2 over reserve 1.