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This document addresses the following Frequently Asked Questions about info-gap decision theory. This document is not a comprehensive presentation of info-gap theory, but does contain references to some literature. Many more literature citations are found elsewhere on this site.

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1 Does an Info-Gap Model only Deal with Local Uncertainty?

Question:

> The best estimate, $\widetilde{u}$, of an info-gap model of uncertainty is sometimes a wild guess, since in most cases the horizon of uncertainty, α, is unknown. How sure can we be that an info-gap model of uncertainty $\mathcal{U}(\alpha, \widetilde{u})$ is not just a local analysis of risks which grossly errs in the true value u? Is it not preferable to employ qualitative methods for managing "unknown-unknowns"? Does the info-gap approach simply sweep major risks under the carpet?

Explanation.

1. Info-gap theory is useful precisely in those situations where our best models and data are highly uncertain, especially when the horizon of uncertainty is unknown. In contrast, if we have good understanding of the system then we don't need info-gap theory, and can use probability theory or even completely deterministic models. It is when we face severe Knightian uncertainty that we need info-gap theory.

2. An info-gap analysis is not based on an estimate of the true horizon of uncertainty. That is, the info-gap model of uncertainty is **not** a single set, $\mathcal{U}(\alpha, \widetilde{u})$. Rather, an info-gap model is a family of nested sets, $\mathcal{U}(\alpha, \widetilde{u})$ for all $\alpha \geq 0$. The family of sets is usually unbounded.[1] Thus an info-gap model is not a "local analysis of risk" since the family of sets expands, usually boundlessly, as the unknown horizon of uncertainty, α, grows. Info-gap theory is **not** a worst-case analysis, since there is no known worst case in an info-gap model of uncertainty.

3. The elements of an info-gap model of uncertainty can be scalars, or vectors, or functions (e.g. constitutive relations or pdf's), or even sets of such entities. As such, an info-gap model is a very flexible tool for non-probabilistically quantifying Knightian uncertainty.

4. Since we indeed do not know how wrong our data and models are (we don't know the true value of α) we evaluate a proposed design by asking: what is the greatest horizon of uncertainty at which the design will still yield acceptable results? The answer to this question is the robustness function. The robustness function generates a preference ordering on the available designs: a more robust design is preferred over a less robust design.

5. The best guess (of models, data, and understanding), around which the info-gap model of uncertainty expands, does influence the info-gap analysis. It is usually true that a change in this best guess will alter the decision which is made. This does not mean, however, that we assume that the best guess is correct. On the contrary, the info-gap robustness analysis addresses the question: how wrong can our best guess be, and the contemplated decision still yields adequate results?

6. Info-gap theory proposes an additional decision function for evaluating proposed designs: the opportuneness function. Since we expect that our models err substantially, we can ask: how wrong must our models and data be in order to enable (though not guarantee) a wonderful windfall outcome, much better than anticipated? The opportuneness function provides additional insight into the implications of the severe uncertainty, helping to choose designs which are able to exploit favorable contingencies. Since designers are usually (and rightly) risk averse, they usually focus primarily on the robustness function. Nonetheless the opportuneness function provides important insight.

7. Value judgments, such as used in qualitative methods, are very important when dealing with severe uncertainty. An important value judgment is: how robust is robust enough? Info-gap theory does not preclude or exclude such value judgments. On the contrary, info-gap theory and the robustness function (as well as the opportuneness function) underlies and supports the value judgments which are needed in making decisions. Info-gap analysis helps one to explore and understand the relation between what we know, what we don't know (including Mr. Rumsfeld's famous "unknown-unknowns"), and the design options which we have. Value judgments are needed

[1]The family of sets is bounded only when there is a physical or definitional limit of the range of variation, such as probabilities not being larger than unity, or masses not being negative.

to make a final integration. Value judgments are discussed at great length in chapter 4 of *Info-Gap Decision Theory*,[2] though that is by no means a conclusive treatment of the relation between qualitative and quantitative methods.

[2]Yakov Ben-Haim, 2006, *Info-Gap Decision Theory: Decisions Under Severe Uncertainty,* 2nd edition, Academic Press, London.

2 Are Info-Gap Models of Uncertainty Based on the Principle of Ignorance?

Question:

> Are info-gap models of uncertainty based on the principle of ignorance (also known as the principle of insufficient reason, or maximum entropy)? Do info-gap models implicitly assume a uniform probability distribution?

Explanation.

The Principle of Insufficient Reason (also called the principle of ignorance, or more recently the principle of maximum entropy) asserts that, in the absence of knowledge about fundamental events, one is entitled to ascribe equal probability to these fundamental events. The principle of insufficient reason leads people to model "ignorance" with a uniform probability distribution. However, when used incautiously, this can lead to paradox, as discussed in a chapter of Keynes' *Treatise on Probability,*[3] which deals with this principle. One of Keynes' examples, as well as others, are discussed in section 2.2 of *Info-Gap Decision Theory.*[4]

This question is sometimes motivated by the non-probabilistic nature of an info-gap model of uncertainty. However, it is incorrect to conclude that info-gap theory assumes a uniform distribution on the elements of the sets of an info-gap model. An info-gap model entails no statement about the likelihood of these elements. An info-gap model is less informative than any probability distribution, even the uniform distribution. It is a common error to think that the uniform distribution expresses maximum ignorance or maximum uncertainty. An info-gap model of uncertainty is less informative than any probability distribution, because an info-gap model is consistent with an infinity of different probability distributions, and does not distinguish between them. This error is common among those who are used to thinking about uncertainty purely in probabilistic terms.

[3] John Maynard Keynes, 1929, *Treatise on Probability,* Macmillan and Co.

[4] Yakov Ben-Haim, 2006, *Info-Gap Decision Theory: Decisions Under Severe Uncertainty,* 2nd edition, Academic Press, London. Related examples are discussed in:
Yakov Ben-Haim, 1996, *Robust Reliability in the Mechanical Sciences,* Springer-Verlag, Berlin.

3 Is Robust-Satisficing the Same as Max-Min?

Question:

> Is info-gap theory simply a re-invention of the max-min principle? Does info-gap robust-satisficing lead to the same decisions as a max-min decision strategy?

Explanation.

Info-gap robust-satisficing and the max-min decision strategy are not the same, but there is a close relation between them.[5] The basic argument developed here is this:

- If max-min and info-gap robust-satisficing are identical decision strategies, then the decisions made under both strategies should always be the same.
- Max-min and robust-satisficing decisions are *not* always the same (which is illustrated below).
- Hence max-min and info-gap robust-satisficing are not identical decision strategies.

I will now explain this argument.[6]

1. Our task is to make a decision, d, from which the reward will be $R(d, u)$, which depends on the uncertain value of u. The first thing to note is an important trade-off: as the uncertainty increases, the minimum reward which could result from the decision becomes smaller. This is an elementary result in max-min theory and in info-gap theory, and is illustrated in fig. 1. In info-gap theory these are called "robustness curves".

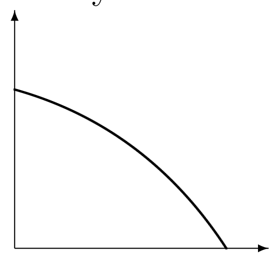

Figure 1: Trade-off between uncertainty and minimum reward.

2. Now suppose that we know, or have estimated, the horizon of uncertainty. This determines the minimum reward which can result from decision d, as illustrated in fig. 2 on p.6.

[5] Further development of these ideas can be found in:
∘ Yakov Ben-Haim, 2007, Robust-satisficing and the probability of survival, working paper, section 9.
∘ Y. Ben-Haim, Akram, Q.F., and O. Eitrheim, 2007, Comparison of min-max and robust-satisficing for managing uncertainty in monetary policy, Working paper.

[6] Max-min and min-max are identical decision strategies, where the former applies when dealing with "reward" (for which large values are desired), while the latter applies when dealing with "penalty" (for which small values are desired). Info-gap robust-satisficing can be applied in both cases.

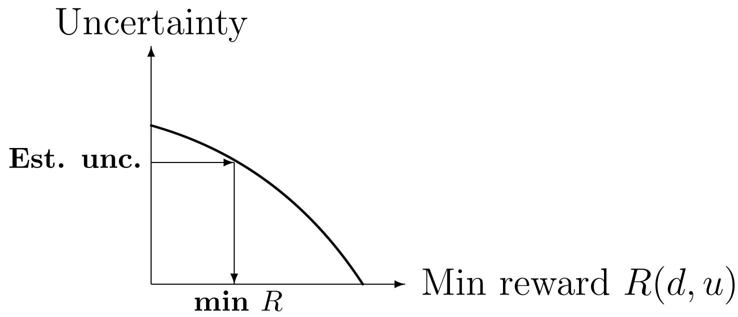

Figure 2: Minimum reward resulting from known or estimated horizon of uncertainty.

3. Now consider the choice between two decisions, d_1, d_2, assuming that we know the horizon of uncertainty. Their trade-off curves, which are assumed to cross,[7] are shown in fig. 3, which also shows that the max-min choice is decision 1: this is the choice which maximizes the minimum reward.

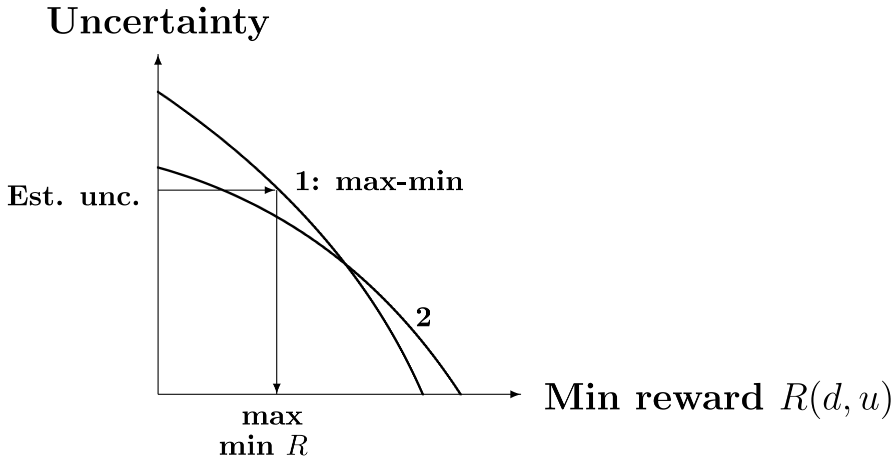

Figure 3: Choosing between two decisions based on max-min.

4. But suppose that the decision maker does not know an estimate of the horizon of uncertainty; does not know how wrong, different, or suprising things can be. When our understanding of typical cases is highly incomplete or uncertain, we are even more unsure how atypical things can be. In this case, the decision maker is unable to apply the max-min algorithm. Robust-satisficing may still an option. It may still be possible for the agent to identify the lowest acceptable or tolerable reward. Denote the lowest acceptable reward as the "Critical R", R_c. For instance R_c might be a break-even reward required by fiduciary policy, or a statutory or stake-holder requirement. Fig. 4 shows the robust-satisficing decision in this case for which a max-min decision is not available at all.

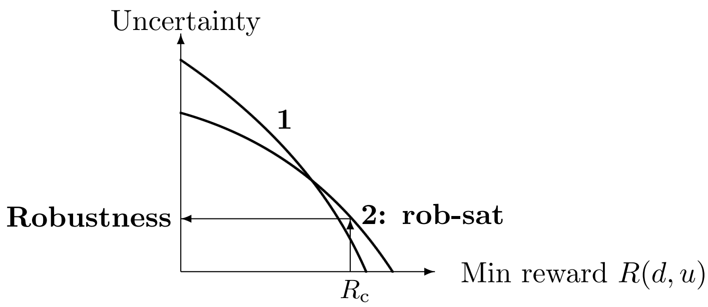

Figure 4: Robust-satisficing choice.

[7]Crossing robustness curves occur frequently, but do not necessarily occur for all pairs of decisions.

5. Let us again suppose that the decision maker has estimated the horizon of uncertainty. Consider a situation in which the minimum reward obtained with the max-min strategy is less than the lowest acceptable reward. That is, an acceptable outcome cannot be guaranteed under the acknowledged level of uncertainty. This is a very common situation; we very often recognize that the ambient uncertainty entails the possibility of unacceptable outcome. Reward less than R_c is unacceptable and neither d_1 nor d_2 can be relied upon to guarantee reward no less than R_c.

We see in fig. 5 that, when requiring reward no less than R_c, the more robust of the two actions is d_2, not the max-min choice d_1. The robust-satisficing strategy is to select d_2 over d_1 since, in this way, we choose the action which will yield the required outcome for a larger range of contingencies. The robust-satisficing decision maker will choose d_2 over d_1 even if he knows that the horizon of uncertainty is as large as indicated in the figure. He does this because the greater robustness of d_2, at the critical reward, implies greater confidence in an acceptable reward.

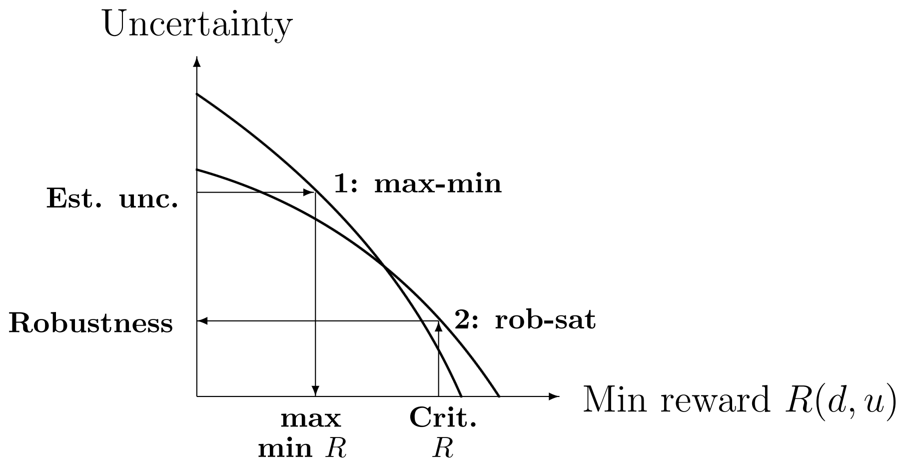

Figure 5: Comparing the robust-satisficing and max-min choices.

6. We have now completed our demonstration that the max-min and robust-satisficing strategies can lead to different decisions, even under the same prior information (knowing the horizon of uncertainty). Obviously, if the decision maker does not know the horizon of uncertainty then he cannot implement the max-min strategy at all. But even if he does know the horizon of uncertainty, then he might not choose the max-min option, depending on his requirement, R_c, regarding the outcome. Of course, the max-min and robust-satisficing strategies will lead to the same decision if the decision maker's required outcome, R_c, is below the crossing point of the robustness curves (or if the robustness curves do not cross at all). However, it is important to note that, in this case, the max-min-er and robust-satisficer choose the same action, but for different reasons. The conclusion of the syllogism is: max-min and info-gap robust-satisficing are not identical decision strategies.

Further motivation for robust-satisficing over max-min-ning is provided by the "proxy theorems" discussed in question 5 on p. 9.

4 Can the Max-Min Strategy be Used to Describe Robust-Satisficing Behavior?

Question: Can the max-min strategy always be used to describe robust-satisficing behavior?

Explanation.

The short answer is "yes", the max-min strategy can always be used to explain or describe or model the behavior of a robust-satisficing decision maker, and vice versa. This of course does not imply that decisions based on the max-min and the robust-satisficing strategies will always be the same. In fact, they may differ, as explained in question 3. I will now briefly describe the **modeller's equivalence** between max-min and info-gap robust-satisficing.[8]

A major source of confusion between the max-min and robust-satisficing strategies arises from the fact that one can always use the max-min theory to describe the behavior of a robust-satisficing decision maker (and vice versa). Looking again at fig. 5 on page 7, suppose we have observed a decision maker to have chosen d_2 over d_1. We can explain this behaviour by (1) supposing that this decision maker thinks that the horizon of uncertainty is the value indicated by "Robustness" on the vertical axis and (2) supposing that the decision maker has used the max-min strategy. In fact, we can always describe robust-satisficing behaviour as though it were max-min behaviour, simply by supposing that the decision maker believes in a suitably chosen horizon of uncertainty. Thus academic researchers, such as many economists, are able to successfully explain the behaviour of decision makers with the max-min theory by supposing those decision makers to have particular beliefs.

However, decision makers themselves do not have the freedom to adjust their beliefs about the horizon of uncertainty so as to make the max-min outcome acceptable. If they have beliefs about the horizon of uncertainty, such as the value indicated by "Est. unc." on the vertical axis of fig. 5, then this leads prediction of the minimum possible reward. If this minimum reward is unacceptably low, then they cannot (or at least should not) adapt their beliefs just so that the max-min strategy indicates an acceptably large minimum reward. In particular, if the decision maker's beliefs are such that the max-min strategy could lead to an unacceptable outcome, then the decision maker is motivated to use the robust-satisficing strategy because robust-satisficing is more robust than max-min-ning.[9]

[8]Further development of these ideas is found in:
Yakov Ben-Haim, 2007, Robust-satisficing and the probability of survival, working paper.

[9]This argument is expanded in response is question 3 on p. 5.

5 Does Maximum Robustness Imply Maximum Likelihood of Success?

Question: Does maximum robustness imply maximum likelihood of success? Even though an info-gap model of uncertainty is non-probabilistic, does the info-gap robustness function nonetheless reveal something about the underlying probability?

Explanation. Yes, in specific conditions, any change in the decision which increases the info-gap robustness also increases the likelihood that the outcome will be acceptable. In other words, robustness is a proxy for the probability of success. The **proxy theorems** which are now discussed provide decision makers with an additional motivation for robust-satisficing.[10]

We must make a decision, d, from which the reward will be $R(d,u)$, which depends on the uncertain value of u. Let $R_{\rm c}$ denote the critical level of reward: the lowest reward which the decision maker is willing or able to accept. Thus "success" or "survival" is the condition:

$$R(d,u) \geq R_{\rm c} \tag{1}$$

If we knew the probability density, $p(u)$, of the uncertain quantity, u, then we could evaluate the probability of survival as:

$$P_{\rm s}(d, R_{\rm c}) = \int_{R(d,u) \geq R_{\rm c}} p(u)\, {\rm d}u \tag{2}$$

However, we are considering Knightian or severe uncertainty, for which we do not know this pdf. However, we do have an info-gap model for uncertainty, which is much less informative than a pdf. Thus we are able to evaluate the robustness, $\widehat{\alpha}(d, R_{\rm c})$. There are various proxy theorems, such as the following, which, under particular conditions, assert that the info-gap robustness is a surrogate for the probability of success:

$$\left(\frac{\partial \widehat{\alpha}(d, R_{\rm c})}{\partial d} \right) \left(\frac{\partial P_{\rm s}(d, R_{\rm c})}{\partial d} \right) \geq 0 \tag{3}$$

This relation asserts that any change in the decision, d, (which in eq.(3) is assumed to be a scalar) which augments the robustness, also augments the probability of success, and vice versa. This implies that, by choosing the decision to maximize the robustness at the required level of outcome, $R_{\rm c}$, we also maximize the probability of achieving that outcome.[11] While we will not know the value of that probability (and it might be very small) we are nonetheless able to maximize it. We see that, when the proxy theorem holds, the robust-satisficing decision is more likely to yield an acceptable outcome than the max-min decision, since the robust-satisficing decision has greater robustness. This emphasizes again the difference between the max-min and robust-satisficing strategies (see question 3 on p. 5), and explains why real decision makers often prefer robust-satisficing over max-min (as distinct from some academics who don't actually make decisions but instead develop models for describing how decision makers behave). Several examples of the info-gap analysis of robust-satisficing behavior—including the Ellsberg and Allais paradoxes, and the equity premium puzzle—have been published.[12]

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[10] Further development of these ideas is found in:

∘ Yakov Ben-Haim, 2006, *Info-Gap Decision Theory: Decisions Under Severe Uncertainty,* 2nd edition, Academic Press, London, section 11.4.

∘ Yakov Ben-Haim, 2007, Robust-satisficing and the probability of survival, working paper.

[11] All local maxima of $\widehat{\alpha}$ and $P_{\rm s}$ coincide. The problem of local vs. global maximization remains.

[12] Yakov Ben-Haim, 2006, *Info-Gap Decision Theory: Decisions Under Severe Uncertainty,* 2nd edition, Academic Press, London, chapter 11.

6 Can Max-Min Computational Tools be Used for Info-Gap Robustness?

Question: A lot of effort in statistics goes into finding methods for determining max-min strategies and estimators. Can these tools be used for calculating info-gap robustness functions?

Explanation. Yes, there is a strong algorithmic relation between max-min and info-gap robustness. In many cases, a technique for finding a max-min property (for some horizon of uncertainty) is equivalent to a technique for finding the info-gap robustness (for some R_c). So algorithms for finding max-min strategies can be used to find info-gap strategies.[13]

[13] See for instance:

Z. Ben-Haim and Y. C. Eldar, Blind minimax estimation, *IEEE Trans. Information Theory,* vol.53, no.9, September 2007, pp. 3145–3157.

Z. Ben-Haim and Y. C. Eldar, Maximum set estimators with bounded estimation error, *IEEE Trans. Signal Processing,* vol.53, no.8, August 2005, pp. 3172–3182.

7 Can Info-Gap Theory Deal with Multiple Performance Requirements?

Question: Can info-gap theory deal with multiple performance requirements, such as multiple requirements like eq.(1) on p.9?

Explanation. Yes. Many examples are found in the literature. Briefly, info-gap robustness (and opportuneness) functions for individual requirements can be combined to deal with multiple simultaneous requirements.