Policy Targeting

Yakov Ben-Haim
Technion—Israel Institute of Technology
yakov@technion.ac.il, info-gap.com

The problem: a decision maker wishes to choose a policy which will guide a system towards a target value. However, the model of the system is highly uncertain. We will illustrate a very simple info-gap analysis of this problem. In addition, we will show that policies which, based on best-estimated model would seem to optimize the outcome, should sometimes be avoided in favor of other policies. We will consider a very simple stylized example. More realistic examples are discussed in the references cited below.

Consider the system model:

$$y = ax + b \tag{1}$$

where a and b are both highly uncertain, with best-estimates $\widetilde{a}$ and $\widetilde{b}$, respectively. The decision maker wishes to choose x so as to pilot y towards a target value, $y^\star$.

The system model, eq.(1), is highly uncertain and we do not know what probability distribution describes the error in the estimates $\widetilde{a}$ and $\widetilde{b}$. The most we can say is that the fractional error in these estimates is unknown. That is, true (or truer) values a and b deviate from the estimated values $\widetilde{a}$ and $\widetilde{b}$ by no more than a fraction h. However, the horizon of uncertainty h is unknown. An info-gap model for this uncertainty is the following unbounded family of nested sets of a and b values:

$$\mathcal{U}(h) = \left\{ a, b : \ \left| \frac{a - \widetilde{a}}{\widetilde{a}} \right| \le h, \ \left| \frac{b - \widetilde{b}}{\widetilde{b}} \right| \le h \right\}, \quad h \ge 0 \tag{2}$$

At any horizon of uncertainty, h, the estimates $\widetilde{a}$ and $\widetilde{b}$ may err fractionally by as much as h. However, the value of h is not known. Thus an info-gap model does not allow a worst case analysis: there is no known worst case since the horizon of error is unknown. We are deep in the domain of Knightian uncertainty.

The performance function is the squared difference between the desired value $y^\star$ and the realized value y:

$$f(x, a, b) = [y(x, a, b) - y^\star]^2 \tag{3}$$

In the spirit of Simon's bounded rationality and the concept of satisficing, we desire the targeting error, $f(x, a, b)$, to be no greater than the critical value $E_{\rm c}^2$:

$$f(x, a, b) \le E_{\rm c}^2 \tag{4}$$

$E_{\rm c}^2$ can be chosen to be small or large to express demanding or modest performance aspirations.

The robustness of policy choice x is the greatest fractional error in the estimates $\widetilde{a}$ and $\widetilde{b}$, up to which every realization a and b results in acceptable squared error. Formally, the robustness of decision x with aspiration $E_{\rm c}$ is:

$$\widehat{h}(x, E_{\rm c}) = \max \left\{ h : \ \left(\max_{a,b \in \mathcal{U}(h)} f(x, a, b) \right) \le E_{\rm c}^2 \right\} \tag{5}$$

Large robustness $\widehat{h}(x, E_{\rm c})$ implies that policy choice x is immune to error in the estimated model while satisficing the targeting-error at $E_{\rm c}$. Low robustness implies that targeting-error as small as $E_{\rm c}$ cannot be confidently expected with choice x.

Let $\widetilde{y}(x) = \widetilde{a}x + \widetilde{b}$ denote the best estimate of the outcome, given choice x, and let us consider values of x for which $\widetilde{y}(x) \le y^\star$. We will assume that $\widetilde{a} > 0$ and $\widetilde{b} > 0$. The robustness of choice x is found to be:

$$\widehat{h}(x, E_{\rm c}) = \begin{cases} \dfrac{E_{\rm c} - [y^\star - \widetilde{y}(x)]}{\widetilde{y}(x)} & \text{if } E_{\rm c} \ge y^\star - \widetilde{y}(x) \\ 0 & \text{else} \end{cases} \tag{6}$$

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Targeting-error no greater than E_c is guaranteed with policy choice x if the horizon of uncertainty is no larger than $\widehat{h}(x, E_c)$.

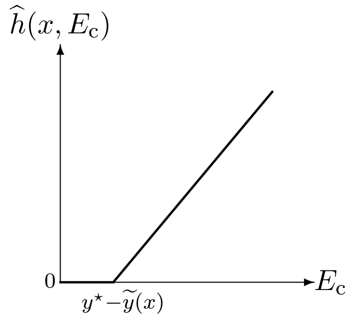

Figure 1: Schematic illustration of the robustness function $\widehat{h}(x, E_c)$ in eq.(6).

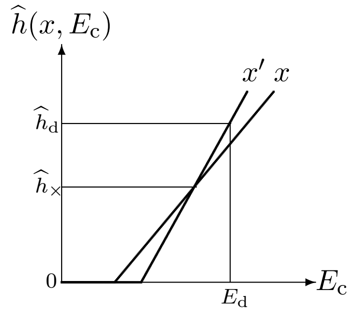

Figure 2: Comparison of two policy choices: reversal of preferences.

As illustrated in fig. 1, the robustness gets worse ($\widehat{h}$ decreases) as the aspired targeting error improves (E_c gets smaller). That is, robustness trades-off against performance.

Furthermore we see in eq.(6) and fig. 1 that the robustness vanishes when the aspiration E_c equals or is less than the best-estimate of the output error, $y^\star - \widetilde{y}(x)$. This is true for any choice of x. We can have little confidence in attaining fidelity as good as the best-estimated fidelity; only poorer fidelity has positive robustness. Since this is true for any x, it is also true for the choice of x which minimizes the estimated error, $f(x, \widetilde{a}, \widetilde{b})$.

This suggests that policies which optimize the outcome should sometimes be avoided.

Let us now consider two policy alternatives, x and x', where:

$$\widetilde{y}(x) > \widetilde{y}(x') > 0 \tag{7}$$

Note that, if $\widetilde{a} > 0$, then eq.(7) implies that $x' < x$. We will say that x' is a "less aggressive" intervention that x.

In particular, let x be the policy choice which, based on the best-estimated model, causes the outcome to precisely match the required value: $\widetilde{y}(x) = y^\star$. This choice of x is what would normally be called the optimal policy. Eq.(7) means that the estimated fidelity is worse with the less aggressive policy x' than with x:

$$0 = y^\star - \widetilde{y}(x) < y^\star - \widetilde{y}(x') \tag{8}$$

The robustness curves for choices x and x' are shown in fig. 2. $\widehat{h}(x', E_c)$ intersects the E_c-axis to the right of $\widehat{h}(x, E_c)$ because the best-estimated fidelity of x' is poorer than for x, eq.(8). However, eqs.(6) and (7) imply that the slope of $\widehat{h}(x', E_c)$ is steeper than the slope of $\widehat{h}(x, E_c)$, so these robustness curves cross.

Crossing of robustness curves implies reversal of preference between choices x and x', where x' is less aggressive than x. Let us suppose that the value of robustness, $\widehat{h}_\times$, at which the curves in fig. 2 cross is fairly low. If we are quite confident that the estimates $\widetilde{a}$ and $\widetilde{b}$ are accurate, then we don't need much robustness, so $\widehat{h}_\times$ might be enough robustness and we would prefer choice x over choice x'. However, we are considering severe Knightian uncertainty: great error in $\widetilde{a}$ and $\widetilde{b}$ is plausible and we need to choose a policy whose anticipated outcome is both acceptable and reliably achieved. Thus, if outcome-error E_d in fig. 2 is good enough fidelity, or if $\widehat{h}_d$ is great enough robustness, then our robust-satisficing preference is for x' over x. If an acceptable combination (E_d, $\widehat{h}_d$) is not found on the x'-curve, then we need to search for some other choice, x'', whose robustness is adequate at acceptable fidelity. If no such x'' exists, then no acceptable policy choice is available in the current state of knowledge. We either revise our aspirations, or do some data-hunting, or revise our model.

This very simple example has illustrated a info-gap robust-satisficing analysis in which one may prefer a policy which is less aggressive than the nominal optimum. Fig. 2 shows that the optimal choice, x, is less desirable than the sub-optimal and less aggressive choice x' under severe uncertainty, because the latter more reliably yields acceptable outcomes (if $E_{\rm d}$ is adequate).

While crossing of robustness curves as in this example is very common, it is not universal. It can happen that robustness curves do not cross one another, in which case the nominal-optimizing choice will coincide with the robust-satisficing choice. However, caution is still called for in assessing what outcome can be considered reliable. Since the trade-off between robustness and outcomes is universal, the robust-satisficing policy maker will not anticipate (or depend upon) the best-estimated outcome because the robustness of this outcome is zero. Rather, by "migrating up" the robustness curve to an acceptable level of robustness, the analyst finds the corresponding outcome which can reliably be anticipated.

Sources.

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