

25th CIRP Design Conference

Managing technological and economic uncertainties in design of long-term infrastructure projects: An info-gap approach

Yakov Ben-Haim,^{a,*} Xavier Irias,^b Roberts McMullin^c

^a*Yitzhak Moda'i Chair in Technology and Economics, Technion—Israel Institute of Technology, Haifa, Israel.*

^b*Director of Engineering and Construction, East Bay Municipal Utility District, Oakland, CA.*

^c*Associate Engineer, East Bay Municipal Utility District, Oakland, CA.*

*Corresponding author. Tel: +972-4-829-3262. E-mail address: yakov@technion.ac.il

Abstract

Infrastructure for water distribution must operate reliably for many decades. Planners face technological and economic uncertainties. The Net Present Worth (NPW) of a long-term infrastructure project is highly uncertain because of these uncertain variables. We use info-gap decision theory for infrastructure planning to manage these uncertainties. We study the robustness question: how much can our estimates of the uncertain variables err, and the NPW will still be acceptable? The answer is expressed by the info-gap robustness function. Large robustness implies great immunity to uncertainty, while low robustness implies high vulnerability to uncertainty. A plan whose robustness is large is preferred over a plan with low robustness. In other words, the info-gap robustness function prioritizes the alternative plans. We illustrate the planning procedure with long-term planning-analysis for maintenance and replacement of Asbestos Cement (AC) pipes owned by the East Bay Municipal Utility District (EBMUD) in Oakland, California. Our example illustrates the evaluation of alternatives based on robustness against uncertainty in both technological and economic variables.

© 2015 The Authors. Published by Elsevier B.V.

Selection and peer-review under responsibility of the International Scientific Committee of “25th CIRP Design Conference” in the person the Conference Chairs Moshe Shpitalni, Anath Fischer and Gila Molcho.

Keywords: technological uncertainty; economic uncertainty; info-gaps; robustness; long-term planning; water infrastructure;

1. Introduction

Infrastructure for water distribution—pipes, pumps and reservoirs—provides an essential service in densely populated urban areas and must operate reliably for many decades. Infrastructure design, construction and maintenance requires large capital investment. Planners face technological and economic uncertainties. Technological uncertainties are of three kinds. First, the requirement for long reliable operation creates an incentive to use innovative technologies. However, what is new is less well understood, especially for long-term service, and hence may be more uncertain than what is conventional. This “innovation dilemma” creates a major uncertainty in the choice between design alternatives [1]. Second, demands on the system (e.g. flow requirements or land use) in the distant future may differ unexpectedly from current demands. Third, material or mechanical properties may change over time in unanticipated ways. Economic uncertainties facing the long-term infrastructure planner arise primarily from uncertainty in the future cost of financing the infrastructure construction and maintenance.

This paper explores the application of info-gap decision theory [2] for infrastructure planning in the face of these uncertainties. We formulate the Net Present Worth (NPW) of a long-term infrastructure project, depending on uncertain technological and economic variables. The planner requires that the NPW be no less than a critical value, below which the project cannot be justified to the stake holders. However, since critical technological and economic variables are uncer-

tain, our estimate of the NPW is also uncertain. Nonetheless, we are able to answer the robustness question: how much can our estimates of the uncertain variables err, and the NPW will still be acceptable? The answer to this question is expressed by the info-gap robustness function. Large robustness implies great immunity to current uncertainty, while low robustness implies high vulnerability to uncertainty. A design whose robustness is large is preferred over a design with low robustness. In other words, the info-gap robustness function prioritizes the alternatives.

We illustrate the planning procedure with the long-term planning-analysis for maintenance and replacement of Asbestos Cement (AC) pipes owned by the East Bay Municipal Utility District (EBMUD) in Oakland, California [3]. EBMUD owns about 3,840 miles (6,180 km) of water-distribution pipes, including 1,145 miles (1,843 km) of AC pipe. An increase in AC pipe failures in the past 7 years led to a study of corrosion by leaching lime from pipe walls. Several studies indicated the need for long-term replacement of existing pipes and raised the possibility of extending the replacement timeline through modified chemical treatment of the carried water [4]. Our example will illustrate the evaluation of alternatives based on robustness against uncertainty in both technological and economic variables.

2. Basic Models

Nomenclature

A_o :	original AC pipe wall thickness [inches].
A :	estimated degraded wall thickness now [inches].
C_{chem} :	water treatment cost [\$/mile].
C_{fix} :	maintenance cost for old pipe [\$/mile].
C_{pipe} :	pipe replacement cost [\$/mile].
C_{op} :	discounted treatment and maintenance cost [\$/mile].
C_{rep} :	discounted pipe replacement cost [\$/mile].
$C_{\text{tot}}(S_i)$:	total discounted cost for strategy S_i [\$/mile].
d :	pipe diameter [inches].
d_{max} :	maximum pipe diameter [inches].
D_{cr} :	critical wall thickness; less is unreliable [inches].
f_{cr} :	fraction of A_o which defines D_{cr} .
$G_{\text{tot}}(S_i)$:	total inventory discounted cost for strategy S_i [\$].
i :	annual interest rate.
$N(d, r, v)$:	number of miles of pipe of diameter d from region r and vintage v .
N_{reg} :	number of regions.
r :	index of geographical region.
$R_b(S_i)$:	inner wall degradation rate for S_i , [inches/year].
R_b^{hist} :	historical inner wall corrosion rate [inches/year].
	May change in future due to chemical treatment.
R_c :	historical outer wall corrosion rate [inches/year].
	Same in past and future.
S_i :	water treatment strategy.
t_{cr} :	number of years from now to reach D_{cr} [years].
t_{plan} :	number of years (into future) of planning analysis.
$t = 1, 2, \dots, t_{\text{plan}}$:	year index into the future.
t_{start} :	number of years from now until starting water treatment strategy S_i .
v :	vintage year, (year the pipe was installed, e.g. 1985).
y_{now} :	current year (e.g. 2014).
v_{max} :	maximum vintage [years].

Wall thickness. Our analysis of pipe wall thickness is based on [3, 4]. Wall thickness of pipe degrades linearly in time:

$$A = A_o - (R_b^{\text{hist}} + R_c)(y_{\text{now}} - v) \quad (1)$$

A pipe is unreliable and eligible for replacement when the wall thickness reaches a fraction f_{cr} of the original thickness:

$$D_{\text{cr}} = f_{\text{cr}} A_o \quad (2)$$

Future inner degradation may change due to treatment strategy, S_i , so, using eq.(1), the critical thickness is:

$$\begin{aligned} D_{\text{cr}} &= A - [R_b(S_i) + R_c]t_{\text{cr}} \quad (3) \\ &= A_o - (R_b^{\text{hist}} + R_c)(y_{\text{now}} - v) - [R_b(S_i) + R_c]t_{\text{cr}} \quad (4) \end{aligned}$$

t_{cr} is the number of years to reach the critical wall thickness.

Combining eqs.(2) and (4) determines t_{cr} :

$$t_{\text{cr}} = \frac{(1 - f_{\text{cr}})A_o - (R_b^{\text{hist}} + R_c)(y_{\text{now}} - v)}{R_b(S_i) + R_c} \quad (5)$$

Costs. The annual maintenance cost for fixing old pipes, C_{fix} , runs from now up to replacement, t_{cr} , or up to the end of the planning time, t_{plan} , whichever comes first. A typical value of C_{fix} is \$20,000/mile. The replacement cost for new pipe, C_{pipe} , is typically \$2.2M/mile, with a typical lower bound of \$1.3M/mile and a typical upper bound of

\$2.5M/mile. The annual water treatment cost, $C_{\text{chem}}(S_i)$, depends on the treatment strategy S_i , $i = 0, 1$ or 2 . The water treatment cost runs throughout the planning time, t_{plan} , and is applied to the water but calculated on a per-pipe-mile basis for all pipe, regardless of whether a pipe is replaced or not. Typical annual water treatment costs per mile for the three strategies are $C_{\text{chem}}(S_0) = \$165/\text{mile}$, $C_{\text{chem}}(S_1) = \$421/\text{mile}$ and $C_{\text{chem}}(S_2) = \$842/\text{mile}$. Capital costs differ between the strategies: S_0 : \$0, S_1 : \$10,000,000 and S_2 : \$20,000,000.

3. Evaluating Net Present Worth

We first consider 1 mile of a specific pipe, and then consider the entire pipe inventory.

1 mile of a specific pipe. We evaluate the Net Present Worth (NPW) of 1 mile of pipe of a given diameter, d , and from a given region, r , using water treatment strategy S_i . In the next section we consider the info-gap robustness analysis.

Step 1. Calculate t_{cr} with eq.(5) for the pipe diameter, region of interest and vintage.

Step 2. Calculate the NPW, with discount rate i , of the operating costs up to the time of pipe replacement, C_{op} :

$$C_{\text{op}} = \sum_{t=t_{\text{start}}}^{t_{\text{plan}}} \frac{1}{(1+i)^t} C_{\text{chem}} + \sum_{t=1}^{\min[t_{\text{cr}}, t_{\text{plan}}]} \frac{1}{(1+i)^t} C_{\text{fix}} \quad (6)$$

The idea behind each term in the sums in eq.(6) is that if you need to spend C_{chem} or C_{fix} in t years from now, you need less than that now because you can put $\frac{1}{(1+i)^t} C_{\text{chem}}$ or $\frac{1}{(1+i)^t} C_{\text{fix}}$ in the bank now and earn compounded interest at the rate i per year for t years. If i is small then you initially need more money. Hence the NPW is large if i is small.

Eq.(6) shows that the NPW of the operating cost is **small** if t_{cr} is **small** because there are few terms in the equation.

Step 3. Calculate the NPW of the future replacement cost at t_{cr} . If t_{plan} is less than t_{cr} then the replacement cost is zero. If not, the replacement cost is positive. We first define an indicator function that tests which time is greater:

$$\mathcal{I}(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0 \end{cases} \quad (7)$$

Now the discounted replacement cost can be expressed as:

$$C_{\text{rep}} = \mathcal{I}(t_{\text{plan}} - t_{\text{cr}}) \frac{1}{(1+i)^{t_{\text{cr}}}} C_{\text{pipe}} \quad (8)$$

The NPW of pipe replacement is **large** if t_{cr} is **small**. This is the reverse of the situation in eq.(6).

Step 4. Calculate the total discounted cost for strategy S_i on 1 mile of pipe with diameter d from region r with eqs.(6) and (8) and using t_{cr} from eq.(5):

$$C_{\text{tot}}(S_i, d, r, v) = C_{\text{op}} + C_{\text{rep}} \quad (9)$$

The entire pipe inventory. $N(d, r, v)$ is the number of miles of pipe of diameter d from region r and of vintage v .

The grand total NPW of strategy S_i for the entire inventory is:

$$G_{\text{tot}}(S_i) = \sum_{r=1}^{N_{\text{reg}}} \sum_{d=1}^{d_{\text{max}}} \sum_{v=1}^{v_{\text{max}}} N(d, r, v) C_{\text{tot}}(S_i, d, r, v) \quad (10)$$

4. Info-Gap Robustness Analysis

We now formulate the info-gap robustness analysis [2].

4.1 Info-Gap Model of Uncertainty

The degradation rates, R_b^{hist} , R_b and R_c , the annual water treatment cost C_{chem} , the annual fixing cost C_{fix} , the replacement cost per mile, C_{pipe} and the interest rate, i , are all uncertain. We have estimates of these variables, and assessments of the errors of these estimates, based on limited measurements or expert judgment. We have no reliable knowledge of maximal deviations from these estimates, and no probability distributions. We represent these uncertainties with the following fractional-error info-gap model:

$$\mathcal{U}(h) = \left\{ \begin{array}{l} R_b^{\text{hist}}, R_b, R_c, C_{\text{chem}}, C_{\text{fix}}, C_{\text{pipe}}, i : \\ \left| \frac{R_b^{\text{hist}} - \tilde{R}_b^{\text{hist}}}{w_b^{\text{hist}}} \right| \leq h, \quad \left| \frac{R_b - \tilde{R}_b(S_i)}{w_b(S_i)} \right| \leq h, \\ \left| \frac{R_c - \tilde{R}_c}{w_c} \right| \leq h, \quad \left| \frac{C_{\text{chem}} - \tilde{C}_{\text{chem}}(S_i)}{w_{\text{chem}}(S_i)} \right| \leq h, \\ \left| \frac{C_{\text{fix}} - \tilde{C}_{\text{fix}}}{w_{\text{fix}}} \right| \leq h, \\ \tilde{C}_{\text{pipe}} - hw_{\text{pipe},1} \leq C_{\text{pipe}} \leq \tilde{C}_{\text{pipe}} + hw_{\text{pipe},2} \\ i \geq 0, \quad \left| \frac{i - \tilde{i}}{w_i} \right| \leq h, \end{array} \right\}, \quad h \geq 0 \quad (11)$$

This info-gap model allows negative degradation rates.

Explanations of the uncertainty terms in eq.(11).

R_b^{hist} and $\tilde{R}_b^{\text{hist}}$ are, respectively, the *unknown true* and the *known estimated* historical values of inner-diameter degradation rate, before chemical intervention. w_b^{hist} is the known error estimate of $\tilde{R}_b^{\text{hist}}$, taken as a typical variation of observed rates. The inequality expresses the unknown fractional error of the estimate, bounded at horizon of uncertainty h . The value of h is unknown: we don't know how much $\tilde{R}_b^{\text{hist}}$ errs.

The R_b , R_c , C_{chem} and C_{fix} uncertainty terms in eq.(11) are likewise unknown fractional errors.

The uncertainty term for C_{pipe} is asymmetric with respect to its estimated value, \tilde{C}_{pipe} , unlike the previous terms which are symmetrically uncertain.

i is symmetrically uncertain around \tilde{i} with the constraint that the interest rate cannot be negative.

4.2 System model, performance requirement and robustness

The **system model** for strategy S_i is the grand total NPW of the inventory, $G_{\text{tot}}(S_i)$ in eq.(10). This depends on all the info-gap uncertain variables, and thus is highly uncertain.

The **performance requirement** is that the grand total NPW not exceed a critical value:

$$G_{\text{tot}}(S_i) \leq G_c \quad (12)$$

The **robustness** of strategy S_i is the greatest horizon of uncertainty, h , in the unknown variables up to which the performance requirement is satisfied:

$$\hat{h}(G_c, S_i) = \max \left\{ h : \left(\max_{(R_b^{\text{hist}}, R_b, R_c, C_{\text{chem}}, C_{\text{fix}}, C_{\text{pipe}}, i) \in \mathcal{U}(h)} G_{\text{tot}}(S_i) \right) \leq G_c \right\} \quad (13)$$

4.3 Evaluating the robustness

Inverse of robustness function. Let $m(h)$ denote the inner maximum in the definition of the robustness, eq.(13). Thus we can re-write eq.(13) more concisely as:

$$\hat{h}(G_c, S_i) = \max \{ h : m(h) \leq G_c \} \quad (14)$$

$m(h)$ increases as h increases (because $m(h)$ is the maximum on the set $\mathcal{U}(h)$ which grows as h increases). Hence a plot of h vs $m(h)$ is identical to a plot of $\hat{h}(G_c, S_i)$ vs G_c . In other words, $m(h)$ is the inverse function of $\hat{h}(G_c, S_i)$.

Calculating $m(h)$. Our basic equations are eqs.(9) and (10). We must consider 7 uncertain variables: R_b^{hist} , R_b , R_c , C_{chem} , C_{fix} , C_{pipe} and i . The last four are easy to handle, the first 3 require a bit more work.

Consider C_{chem} , C_{fix} , C_{pipe} and i first. Each term in eq.(10) is maximized by choosing:

- i as small as possible at horizon of uncertainty h :

$$i = \max[0, \tilde{i} - w_i h] \quad (15)$$

- C_{chem} , C_{fix} and C_{pipe} as large as possible at horizon of uncertainty h :

$$C_{\text{chem}} = \tilde{C}_{\text{chem}} + w_{\text{chem}} h \quad (16)$$

$$C_{\text{fix}} = \tilde{C}_{\text{fix}} + w_{\text{fix}} h \quad (17)$$

$$C_{\text{pipe}} = \tilde{C}_{\text{pipe}} + hw_{\text{pipe},2} \quad (18)$$

Now consider how R_b^{hist} , R_b and R_c influence $m(h)$. It is only the sums, $R_b + R_c$ and $R_b^{\text{hist}} + R_c$, that are important, and they act only through t_{cr} . From eq.(5), a large value of either sum causes a small value of t_{cr} . However, t_{cr} acts in two opposing directions. A large value of t_{cr} causes:

- C_{op} to be large (see eq.(6)).
- C_{rep} to be small (see eq.(8)).

Thus how $R_b + R_c$ and $R_b^{\text{hist}} + R_c$ should be chosen to produce the inner maximum, $m(h)$, must be found by numerically evaluating G_{tot} with:

- The maximizing values of C_{chem} , C_{fix} , C_{pipe} and i , eqs.(15)–(18).
- All combinations of successive small increments of R_b^{hist} ,

R_b and R_c , each chosen from its uncertainty interval at horizon of uncertainty h . These uncertainty intervals are:

$$\tilde{R}_b - w_b h \leq R_b \leq \tilde{R}_b + w_b h \quad (19)$$

$$\tilde{R}_b^{\text{hist}} - w_b^{\text{hist}} h \leq R_b^{\text{hist}} \leq \tilde{R}_b^{\text{hist}} + w_b^{\text{hist}} h \quad (20)$$

$$\tilde{R}_c - w_c h \leq R_c \leq \tilde{R}_c + w_c h \quad (21)$$

The value of $m(h)$ is the maximum value of G_{tot} over this range of R_b^{hist} , R_b and R_c values.

5. Numerical Examples

We present numerical examples of the robustness function and related variables to illustrate the types of results one expects to obtain, how to interpret them, and the types of conclusions that one can reach. While these results are based on plausible values of the parameters, the calculations are made for a single typical 1-mile section of pipe, and not for the full pipe inventory. These results are not design recommendations.

We will evaluate the robustness function as formulated in eq.(13) and based on the system model in eqs.(9) and (10) and the info-gap model in eq.(11). However, since we are considering only 1 mile of a specific pipe, we are in effect assuming: $N_{\text{reg}} = d_{\text{max}} = v_{\text{max}} = N(d, r, v) = 1$.

We first consider low estimated water treatment costs: $\tilde{C}_{\text{chem}}(S_0) = \165 , $\tilde{C}_{\text{chem}}(S_1) = \421 and $\tilde{C}_{\text{chem}}(S_2) = \842 . We then consider high estimated water treatment costs: $\tilde{C}_{\text{chem}}(S_0) = \165 , $\tilde{C}_{\text{chem}}(S_1) = \$4,210$ and $\tilde{C}_{\text{chem}}(S_2) = \$8,420$. These high water treatment costs are meant to roughly represent the capital costs of chemical treatment. Hence, since treatment S_0 has no capital costs, it has the same value for “low” and for “high” costs.

All other variables take the following “standard” values unless otherwise indicated.

Planning variables: $f_{\text{cr}} = 0.2$, $A_o = 0.6$, $t_{\text{plan}} = 30$, $t_{\text{start}} = 4$, $y_{\text{now}} = 2014$, $v = 1985$.

Degradation rates: $\tilde{R}_b(S_0) = 0.0075$, $\tilde{R}_b(S_1) = 0.0053$, $\tilde{R}_b(S_2) = 0.002$, $w_b(S_i) = 0.5\tilde{R}_b(S_i)$, $\tilde{R}_b^{\text{hist}} = 0.0038$, $w_b^{\text{hist}} = 0.5\tilde{R}_b^{\text{hist}}$, $\tilde{R}_c = 0.003$, $w_c = 0.5\tilde{R}_c$.

Costs: $w_{\text{chem}}(S_i) = \$0.3\tilde{C}_{\text{chem}}(S_i)$, $\tilde{C}_{\text{fix}} = \$20,000$, $w_{\text{fix}} = \$0.3\tilde{C}_{\text{fix}}$, $\tilde{C}_{\text{pipe}} = \$2,200,000$, $w_{\text{pipe},1} = \$900,000$, $w_{\text{pipe},2} = \$300,000$, $i = 0.03$, $w_i = 0.01$.

The water treatment strategies are color coded as follows in all graphs: S_0 : blue, S_1 : green, and S_2 : red.

5.1 Low water treatment costs

Fig. 1 shows the robustness function, $\hat{h}(G_c, S_i)$, vs. the critical total cost, G_c , for each of the 3 strategies. Note that S_1 (green) and S_2 (red) have nearly the same estimated cost (horizontal intercept), which is less than for S_0 (blue). S_2 is the most robust strategy at low G_c , and the curves converge at larger G_c .

The estimated value of the total discounted cost of each strategy is the value of the horizontal intercept of the corresponding robustness curve in fig. 1. Based on these estimated values, fig. 1 indicates that the analyst would be indifferent

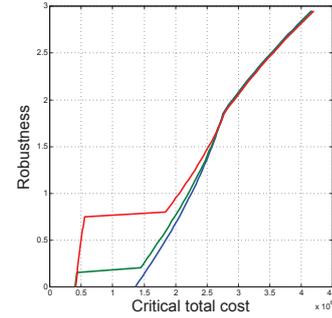


Figure 1: Robustness curves, $\hat{h}(G_c, S_i)$ vs. G_c : low water treatment cost.

between S_1 and S_2 (red and green), and would prefer either of these strategies over S_0 (blue) whose estimated total cost is greater (S_0 's horizontal intercept is further to the right). However, we see in fig. 1 that the robustness for achieving the estimated cost is zero, so estimated cost is not a good basis for choosing a strategy. Rather, one must look at the entire robustness curve. We see that S_2 is more robust than S_1 over a range of G_c values, until the curves gradually converge at higher cost and higher robustness. While S_2 is robust-preferred over the other strategies, the degree of the preference diminishes as the robustness increases, and the value of total cost that can be reliably assigned to S_2 is substantially greater than the estimated value.

5.2 High water treatment costs

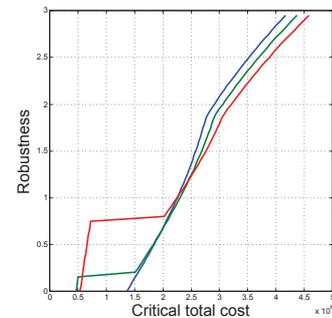


Figure 2: Robustness curves, $\hat{h}(G_c, S_i)$ vs. G_c : high water treatment cost.

Fig. 2 shows the robustness function as in fig. 1, now for high-cost water treatment. The results are similar, but with important differences. For low cost strategies (fig. 1) S_2 was robust-dominant until the curves converge. However, with high cost strategies (fig. 2) we see preference reversals expressed by crossing robustness curves. In fig. 2 S_1 (green) is most robust (and hence preferred) at small (good) G_c (but note that the robustness is low). Then S_2 (red) becomes most robust at intermediate G_c values (and moderate robustness values). Finally S_0 (blue) is most robust at larger G_c and greater robustness. In summary, the preference ordering of the three strategies depends on the required value of

total discounted cost, G_c , and on judgment of how much robustness is needed.

5.3 Other Cases

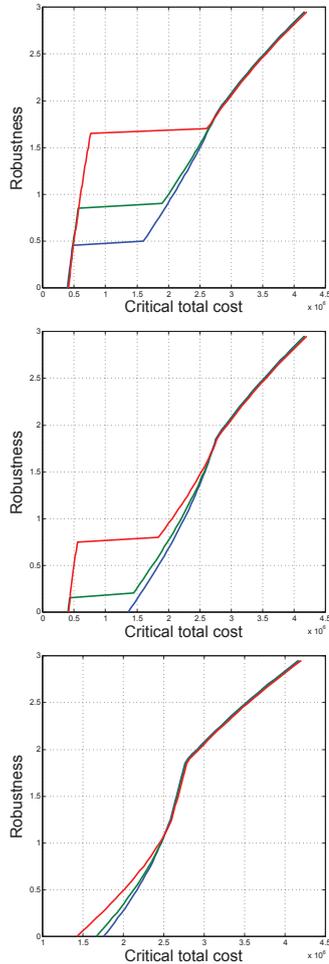


Figure 3: Robustness curves, $\hat{h}(G_c, S_i)$ vs. G_c . Low water treatment cost and “standard” values except $A_o = 0.8$ (top), $A_o = 0.6$ (center, same as in fig. 1) and $A_o = 0.4$ (bottom).

Fig. 3 shows robustness curves at low water treatment cost and with the “standard” values of all variables except A_o which takes the values 0.8, 0.6 and 0.4 inches in the top, center and bottom frames. The center frame is thus the standard case presented in fig. 1. Comparison of these figures shows the substantial impact on the robustness of initial wall thickness, A_o . When $A_o = 0.8$ (top), the 3 strategies are nominally indistinguishable (they have the same horizontal intercepts) but S_2 (red) is significantly robust-preferred. At the other extreme, when $A_o = 0.4$ (bottom), the 3 strategies are roughly indistinguishable both nominally and in terms of robustness.

Fig. 4 shows robustness curves at low water treatment cost and with the “standard” values of all variables except f_{cr} which takes the value 0.3. Comparing fig. 4 with the robustness curves in fig. 1 we see the substantial impact on

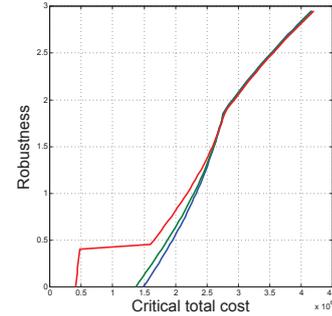


Figure 4: Robustness curves, $\hat{h}(G_c, S_i)$ vs. G_c . Low water treatment cost and “standard” values except $f_{cr} = 0.3$.

the robustness of critical wall thickness, f_{cr} . In fig. 4 strategy S_2 is robust-dominant, though weakly so, and strategies S_0 and S_1 are essentially indistinguishable.

6. Conclusion

We have applied info-gap robustness analysis to the long-term planning of a large water distribution system. Info-gap robustness analysis can be applied to any short-term or long-term asset management program (AMP) or lifecycle cost analysis (LCCA) that evaluates alternatives in attempting to increase efficiency and financial savings. The AMP or LCCA can be scrutinized to every component that contributes to cost. These components are analyzed to understand the level of uncertainty or confidence that the project owner is willing to accept. The components can include recurring costs, capital costs, and even the interest and discount rates. Info-gap robustness analysis provides an approach that can assist planners and decision makers in making recommendations based upon appreciating the limits of one’s knowledge.

References

1. Ben-Haim, Yakov, Craig D. Osteen and L. Joe Moffitt, 2013, Policy Dilemma of Innovation: An Info-Gap Approach, *Ecological Economics*, 85: 130–138.
2. Ben-Haim, Yakov, 2006, *Info-Gap Decision Theory: Decisions Under Severe Uncertainty*, 2nd edition, Academic Press, London.
3. East Bay Municipal Utility District, 2014, Info-Gap Analysis of Water Chemistry & Asbestos Cement Pipe Replacement, Oakland, CA.
4. East Bay Municipal Utility District, 2012, Phase I Asbestos Cement Pipe Corrosion Study, Technical Report, Section 9, July 2012, Oakland, CA.