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Is your profiling strategy robust?

LIOR DAVIDOVITCH†
Faculty of Mechanical Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel

AND

YAKOV BEN-HAIM‡
Yitzhak Moda'i Chair in Technology and Economics, Faculty of Mechanical Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel

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The economic theory of crime views criminals as rational decision makers, implying elastic response to law enforcement. Group-dependent elasticities can be exploited for efficient allocation of enforcement resources. However, profiling can augment both number of arrests and total crime since non-profiled groups will increase their criminality. Elasticities are highly uncertain, so prediction is difficult and uncertainty must be accounted for in designing a profiling strategy. We use info-gap theory for satisficing (not minimizing) total crime rate. Using an empirical example, based on running red lights, we demonstrate the trade-off between robustness to uncertainty and total crime rate.

Keywords: info-gap decision theory; profiling; severe uncertainty; satisficing.

1. Introduction

The modern economic view of crime is traditionally traced back to Becker (1968). In his seminal paper, Becker notes that potential offenders come from various backgrounds and therefore have different responses to the probability of conviction and to the expected punishment. He suggested that parameters such as premeditation, sanity and age may be used as proxies for the offenders' elasticities of response to punishment. The combination of elasticities for different groups, and the ability to statistically predict the elasticities via proxies, is the basis for 'statistical discrimination' (Arrow, 1973) or profiling.[1]

Although profiling has been shown to be potentially beneficial (as a tool for minimizing crime rate or maximizing some abstract social benefit), it has been argued that the inequality that is the essence of profiling is, in fact, unjust. After Lamberth (1994) showed that there is a discriminatory policy, either official or *de facto*, against African American drivers in the context of drug interdiction, much research discussed to what extent profiling can be justified as an economic result, rather than a racial bias (Knowles *et al.*, 2001; Borooah, 2001; Hernández and Knowles, 2004), and what is the

† Email: liordav@technion.ac.il
‡ Corresponding author: Email: yakov@technion.ac.il

[1] Of course, the use of actuarial tools in the context of criminology predates Becker. It is related to Burgess (1928), but he did not refer to profiling as a result of an economic model or suggest an economic model to utilize the differences between the groups.

trade-off between equality and efficiency of law enforcement (Farmer and Terrell, 2001; Persico, 2002; Blumkin and Margalioth, 2005).[2]

Setting aside ethical considerations, profiling has been criticized for being 'inefficient'. Harcourt (2007)[3] demonstrates that, under a set budget, targeting groups with higher crime rates may cause the total crime rate to 'increase'. This is because shifting enforcement resources to a minority group will cause the remaining majority to increase its participation in crime. The net effect can be an increase in both total arrests and total crime. Bearing that in mind, a policy maker who wishes to use profiling as means for reducing the total crime rate must take into account not only the current crime rates of the different groups but also the groups' responsivenesses (or elasticities) to policing. The trade-off between the arrest rate, which represents efficiency in the eyes of the police officer, and the crime rate, which represents efficiency in the eyes of the police force, was also discussed by Alexeev and Leitzel (2002).

However, it is extremely difficult to estimate the responsiveness of crime to policing even for the general population. In fact, many researchers find that the correlation between policing efforts and crime is either non-existent or positive (more policing means more crime).[4] Although it has been argued that the main reason for this is simultaneity problems,[5] it is still non-trivial to estimate the responsiveness to policing.[6] Over the years, researchers have tried to optimize the social welfare (or some proxy of the social welfare) using the responsivenesses of the different groups within the population[7] (Becker 1968; Benson and Bowmaker 2005), the utility to the offender from the illegal act (Malik 1990; Polinsky and Shavell 2000), the dis-utility from disrepute due to conviction (Polinsky and Shavell 2000; Pradiptyo 2007), the knowledge of the criminals regarding their probability of conviction and expected punishment (Polinsky and Shavell 2000) and so on. The huge uncertainties of the models involved were often overlooked. A noted exception is Manski (2006), who tries to eliminate dominated profiling strategies when the responsiveness to policing (deterrence) is unknown, but the set of possible responsiveness functions is known. Bar-Ilan and Sacerdote (2004) point out that estimating the elasticity to fine increases for running red lights may be compounded by 'other costs to receiving a ticket, including increased insurance premiums, time costs and feelings of guilt'. Such effects may be difficult to quantify and can vastly change the estimated elasticities.

[2] Heaton (2006) shows the decreased efficiency of policing (resulted in an increase in crime rates) due to 'anti-profiling' policy implemented in New Jersey.

[3] Similar argument is presented in Harcourt (2004, 2006).

[4] See Levitt (1997) for examples of empirical research. Tsebelis (1990) uses game theoretic reasoning to prove that the crime rate is independent of the severity of the punishment, though it might be influenced by the probability of detection. See also Ehrlich and Liu (1999) as an example for the debate in the question of deterrence. On the other hand, Levitt (1998) reports a strong negative correlation between arrest rates and reported crime rates.

[5] Crime and law enforcement affect each other simultaneously since high crime rates lead to further investment in law enforcement.

[6] Levitt (1997) uses the assumption that the growth in police size during election years (be it mayoral or gubernatorial) is not related to the level of crime to compare the level of crime in election years to non-election years. Although McCrary (2002) has shown the calculation of Levitt (1997) to be flawed, Levitt (2002) argued that many of the results hold even after applying the corrections of McCrary (2002). Levitt (2002) also uses the number of different municipal workers (firefighters, street and highway workers, etc.) to examine the correlation between policing and crime rate. Klick and Tabarrok (2005) utilize the increased police presence in periods of high alert to show that the crime rate is mostly reduced in the area of the National Mall in Washington DC, which is supposed to have a higher presence of police in periods of high alert (this district hosts the White House, Congress, Supreme Court and so forth). Chakravarty (2002) argues that even when detailed data concerning policing efforts and arrest rates are available, Type 1 and Type 2 errors of the policing force make the estimation of the actual crime rates and responsiveness to policing extremely difficult.

[7] Or the supply of offences.

In this paper, we suggest the use of info-gap decision theory (Ben-Haim 2006) for formulating a profiling strategy, whereby one tries to 'satisfice'[8] the total crime rate rather than to 'optimize it'. By 'satisficing', we mean keeping the value of a loss function (like total crime rate) below an acceptable level. Satisficing is to be distinguished from optimizing which entails minimizing the loss. The motivation for satisficing (rather than optimizing) derives from the great uncertainty associated with estimates of the responsiveness to policing. We will demonstrate the irrevocable trade-off between robustness to this uncertainty on the one hand and reduction of the total crime rate on the other hand. An allocation that attempts to minimize total crime is an allocation with zero robustness to uncertainty in the responsiveness function. Under a fixed budget, elastic response to profiling can result in an increase in total crime. Hence, knowledge of the elasticity is critical. When this knowledge is highly uncertain, it is necessary to choose an allocation that is robust to this uncertainty while at the same time aiming at adequate reduction in total crime. Allocation must aim to reliably achieve acceptable reduction—rather than minimization—of the total crime rate. The quantitative analysis of this trade-off underlies the choice of an allocation.

We will demonstrate the profiling of two groups with uncertain responsiveness functions and show how to choose an allocation of police resource, which will be robust to errors in the estimation of responsiveness functions. We will give a numerical example based on research that estimated the elasticities of different groups to policing in the context of driving through red lights (Bar-Ilan and Sacerdote, 2004).

The paper is organized as follows. Section 2 briefly describes how info-gap decision theory is used to robustly satisfice a requirement. Section 3 exemplifies the use of info-gap decision theory in the case of profiling traffic violators. Section 4 expands the analysis to incorporate the dynamics of the population. A concluding discussion appears in Section 5. Mathematical definitions and derivations appear in appendices.

2. Info-gap decision theory: an intuitive discussion

In this section, we present an intuitive description of info-gap models of uncertainty, and how info-gap models can be used for deriving robust decisions.

Decision making may be viewed as choosing a decision q from a set Q of feasible decisions. The outcome of the decision is expressed as a loss, $L(q, u)$, where u is the value of parameters or functions that are unknown or uncertain to the decision maker when the decision was made. u may be, for instance, the parameters of a model, or a functional relationship between variables, or a probability distribution of random variables or sets of such entities. In this paper, u is the uncertain responsiveness to policing. We have a best estimate of u, denoted $\widetilde{u}$, but our uncertainty about u is non-probabilistic. That is, we do not know a probability distribution that describes the uncertainty of u. In many cases, the uncertainty about u is unbounded: we cannot identify a worst case. Our analysis will be based on info-gap decision theory (Ben-Haim, 2006).

Info-gap models are used to quantify non-probabilistic Knightian uncertainty (Ben-Haim, 2006). An info-gap model is an unbounded family of nested sets. At any level of uncertainty, a set contains possible realizations of u. As the horizon of uncertainty gets larger, the sets become more inclusive. The info-gap model expresses the decision maker's beliefs about uncertain variation of u around $\widetilde{u}$.

[8] Etymologically, 'to satisfice' is a variant on 'to satisfy', but satisfice has come to have a tighter technical meaning in economics, psychology and decision theory. The Oxford English Dictionary (2nd edition, 1989) defines satisfice to mean 'To decide on and pursue a course of action that will satisfy the minimum requirements necessary to achieve a particular goal.

Info-gap models of uncertainty obeys two axioms:

1. *Contraction:* $\widetilde{u}$ is the only possibility when there is no uncertainty.
2. *Nesting:* the range of possible realizations increases as the level of uncertainty increases.

Suppose the decision maker wishes to reduce the loss and has some notion of a critical loss L_c, whose exceedence cannot be tolerated. The 'robustness' of a decision q, denoted $\widehat{\alpha}(q, L_c)$, is the greatest level of uncertainty, which still guarantees a loss no greater than L_c. That is, as long as our best estimate $\widetilde{u}$ is erroneous by no more than $\widehat{\alpha}(q, L_c)$, the loss would be acceptable. Note that the 'amount of uncertainty' (or horizon of uncertainty) measures the maximal deviation between the actual state of the world, u, and our best estimate, $\widetilde{u}$.

'Robust-satisficing' decision making maximizes the robustness and keeps the loss less than the value L_c, without specifying a limit on the level of uncertainty. That is, given a critical loss, L_c, the decision maker will choose the decision $\widehat{q}$ with greatest robustness to uncertainty. Under non-probabilistic Knightian uncertainty, this is an attempt to maximize the confidence in achieving no more than an acceptable loss.

It can readily be shown that there is an inherent trade-off between robustness and performance. Since robustness is the 'immunity to failure', the robustness decreases as the performance requirement L_c becomes more demanding. That is, $\widehat{\alpha}(q, L_c)$ gets smaller as L_c gets smaller. Another immediate result is that the estimated 'optimal' result—the minimal loss under our best estimate $\widetilde{u}$—has zero robustness, meaning that a slight deviation from our estimation $\widetilde{u}$ may result in exceeding L_c.

3. Case study: running red lights

Not often do we come across data that may be used to infer the responsiveness to policing of different groups within the population. However, we do have such data for driving through a red light. In the USA, roughly 2000 deaths resulted in 1998 from drivers running red lights (Bar-Ilan and Sacerdote, 2004). It should be noted that most of the enforcement of running red lights is done automatically using cameras (Israel Police, 2007). This means that profiling, in its 'natural' meaning of assigning different probabilities of detection to different groups within the population, is not easily implemented but could be done by varying the density of detectors in different regions.

Drug interdiction on highways is much more relevant for profiling. Since a car search is initiated as a result of suspicion by a police officer, it is quite reasonable to assume that the suspicion is somewhat correlated to the group to which the driver of the car belongs, be it an ethnic group, a socioeconomic group or a cultural group. Indeed, much research has examined the correlation between the ethnicity of the driver and the probability of his car being searched (Lamberth, 1994; Knowles *et al.*, 2001; Borooah, 2001; among many others).

Nonetheless, running red lights is one of the rare cases where information regarding responsivenesses of different groups within the population can be found, while similar information regarding drug interdiction is scarce. Therefore, in order to demonstrate the practical use of info-gap decision theory, we assume that running red lights could be profiled in a similar fashion as drug interdiction. Namely, that policing resources could be allocated arbitrarily between different groups, thus affecting the probability of catching a driver running through a red light. We will then use the data gathered on running red lights to demonstrate the robust-satisficing methodology suggested by Ben-Haim (2006).

3.1 *Responsiveness to policing*

Bar-Ilan and Sacerdote (2004) study running through red lights and use incidents of change in the probability of detection (when traffic cameras are added) and changes in the punishment in the case of detection (when the fine for driving through red lights is increased) to show that the responsiveness to these two factors is quite similar, which suggests some degree of risk neutrality of the drivers. In particular, Bar-Ilan and Sacerdote (2004) use very detailed data collected in Israel to compare the responsiveness and crime rates of different groups within the population, after Israel raised the fine for driving through red lights from 400 shekels ($122) to 1000 shekels ($305) in December 1996. As assumed by Harcourt (2007), the responsivenesses (or elasticities) of the different groups are not necessarily similar: young drivers not only have a higher rate of violations but also have an elasticity, which is significantly higher than the general population; drivers convicted of property crimes have a higher rate of red-light violations, but an elasticity that is similar to that of the general population; non-Jewish drivers have a much lower elasticity than the general population.

In order to calculate the entire curve of responsiveness from limited data, we must assume the general shape of the curve. We will use an info-gap model to represent uncertainty in the shape of this curve. Our best guess is that the responsiveness curve of the ith group has the following form:

$$\widetilde{C}_i = \exp\left(-\gamma_i \frac{b_i}{\pi_i} - \delta_i\right), \tag{1}$$

where $\widetilde{C}_i$ is the average crime rate of the ith group: the number of red light incidents per person in the group per time period of 14 quarters (the length of the period examined by Bar-Ilan and Sacerdote, 2004), γ_i and δ_i are parameters that characterize the responsiveness of the ith group, π_i is the fraction of group i within the general population and b_i is the fraction of the budget allocated to police group i. Thus, π_i and b_i are both between 0 and 1. This is the basis of the profiling: by setting $b_i > \pi_i$, we target group i (the fraction of policing resources allocated to group i is greater than its fraction within the general population).

3.2 *Satisficing the crime rate*

A good group to target is a group, which constitutes a considerable fraction of the population of drivers has a high value of γ_i (high elasticity), and of course, can be easily recognized by a police officer. The group of drivers between the ages 17 and 30 meets all the above criteria. Therefore, we shall concentrate our efforts on profiling this group, where the goal is to reliably reduce the total crime rate.

Following is an intuitive review of the process of robustly satisficing the crime rate. Appendix B gives the mathematical definitions and results.

3.2.1 *Info-gap model of uncertainty.* The exponential model representing the responsivenesses of the different groups to policing, (1), is only a rough estimate; the shape of the curve may be different. The crime rate and responsiveness have been measured by Bar-Ilan and Sacerdote (2004) for a specific allocation, which we shall denote b^0. We may be fairly confident of the crime rate for b^0. However, it is reasonable to suppose that the uncertainty in the responsiveness function grows as the difference between the actual allocation, b, and the reference allocation, b^0, increases.

Since we will be examining 'reallocation' of fixed total policing resources, we shall describe an allocation using the fraction of resources allocated to each group. That is, b_i is the fraction, between 0 and 1, of the resources allocated to group i rather than the absolute amount of resources.

Let $\widetilde{C}$ be a vector of responsiveness functions, representing our best estimate of the responsiveness functions of the different groups. $\widetilde{C}_i$ will be based on the exponential model, (1). We will refer to $\widetilde{C}$ as the 'nominal model' and represent the uncertainty surrounding the actual responsiveness functions using an info-gap model. Let C denote the vector of actual responsiveness functions, which may differ in functional form from the nominal vector $\widetilde{C}$. Our info-gap model, which is defined in Appendix B, assumes that the maximal error in our estimation of the responsiveness functions increases as the allocation deviates from b^0. In other words, for any given horizon of uncertainty, there is an envelope surrounding the nominal model, $\widetilde{C}$. Every model within this envelope is allowed. The 'shape' of the envelope is specified (see Appendix B), but the true 'magnitude' of deviation is unknown. The info-gap model is an unbounded family of such envelopes.

3.2.2 *Robustness.* Let b_y denote the allocation of surveillance resources to the 'young' population of 17–30 years old, and let $b_{\bar{y}}$ denote the allocation to the complementary group. Given some critical crime rate L_c, we can calculate the robustness of any given allocation b. Since $b_y + b_{\bar{y}} = 1$, we can represent an allocation through b_y.

In choosing an allocation, we wish to know how wrong the estimated response functions could be, and the allocation would still result in acceptable total crime rate. The robustness, $\widehat{\alpha}(b_y, L_c)$, of an allocation b_y, is the greatest horizon of uncertainty up to which all realizations of the responsiveness functions, C_i, result in total crime rate not exceeding the critical value L_c. Robust-satisficing decision making maximizes the robustness to a specific critical crime rate L_c, without specifying a limit on the actual level of uncertainty. That is, given a critical crime rate, the decision maker will choose the decision $\widehat{b_y}$ with greatest robustness to uncertainty. Under non-probabilistic Knightian uncertainty, this is an attempt to maximize the confidence in achieving no more than an acceptable crime rate.

Figure 1 illustrates the robustness curves for four different allocations of the policing resources: the 'current' allocation ($b_y = 0.145$, which is equal to the fraction of young in the population and is the allocation b_y^0 measured by Bar-Ilan and Sacerdote, 2004), the 'nominal optimal' allocation (in the sense that it yields the minimal crime rate under the nominal model, $b_y = 0.268$) and two other allocations ($b_y = 0.2$ and $b_y = 0.12$). As expected, all robustness curves are monotonic: robustness increases as the critical crime rate increases (a 'weaker' requirement is 'more robustly' achieved). Also, each curve crosses the horizontal axis at the crime rate yielded by the corresponding allocation under the nominal model.

The definition of robustness implies a vertical robustness curve for the reference allocation, b^0. (We are not concerned with the statistical uncertainty of the observation. Rather, we focus on the uncertainty in the shape of the responsiveness functions as the allocation changes from the current value.) We can understand this as follows: we are certain of the crime rate under the current (observed) allocation, b^0. Therefore, the robustness of that allocation is zero for crime rates less than the current crime rate and infinite for crime rates higher than that crime rate. The infinite robustness of the current allocation appears as a vertical curve at $L_c = 0.050$, the current crime rate. This means that the current allocation is the most robust (at the time of measurement) if the critical crime rate is at least the current crime rate.

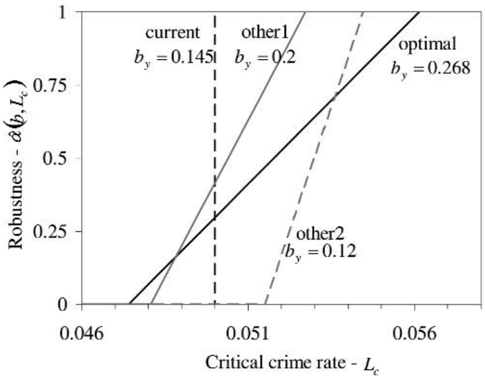

FIG. 1. Robustness curves of four allocations: the current allocation (the allocation measured by Bar-Ilan and Sacerdote, 2004, $b_y = 0.145$); the optimal allocation (the allocation that minimizes the crime rate under the nominal model, $b_y = 0.268$) and some other arbitrary allocations ($b_y = 0.2$ and $b_y = 0.12$). The crossing of the curves indicates a change of preference: for instance, while for critical crime rates no greater than 0.049, the most robust allocation (of the four presented) is the optimal allocation and for critical crime rates between 0.049 and 0.050, the most robust allocation is 'other1'.

The nominal optimal allocation, $b_y = 0.268$, yields the lowest total crime rate under the nominal model. This makes it more robust than any other allocation around the nominal optimal crime rate. However, the optimal crime rate is not a good choice for the critical value since the robustness for the nominal optimal crime rate is zero. This means that the slightest deviation from the assumptions of the models may cause the crime rate to exceed the nominal optimal value.

Note that the nominal optimal robustness curve is crossed by other robustness curves. The crossing of the robustness curve of the nominal optimal allocation means that it is not the most robust allocation for all choices of the critical crime rate. For instance, for crime rates equal or higher than 0.049, the nominal optimal allocation is less robust than the allocation $b_y = 0.2$. Consequently, if a total crime rate of 0.049 (which is lower than the current rate of 0.050) is acceptable, then we would prefer the allocation $b_y = 0.2$ over the allocation $b_y = 0.268$ since the former is more robust than the latter while satisficing the total crime rate at 0.049.

Figure 2 illustrates the correlation between the critical crime rate and the most robust allocation. The most robust allocation, $\widehat{b}(L_c)$, maximizes the robustness and satisfices the total crime rate at the critical value L_c:

$$\widehat{b}(L_c) = \arg\max_{b_y} \widehat{\alpha}(b_y, L_c). \tag{2}$$

An important result is that, with the exception of the current allocation, most allocations are the most robust for only one critical value. The current allocation, which is most robust for any crime rate higher than the current crime rate, stands out as a single exception.

The importance of the above observation to the decision maker is that there is no 'robust-dominant' decision, an allocation which is more robust than any other allocation for all critical crime rates. The most robust allocation is a function of the satisficing criterion, namely, of the crime rate which the policy maker seeks to achieve. In other words, the robust-satisficing allocation, $\widehat{b}(L_c)$, depends on the decision maker's choice of the critical crime rate, L_c. In fact, it can be proved that this is not a coincidental result of a specific choice of model and parameters but rather the general case.

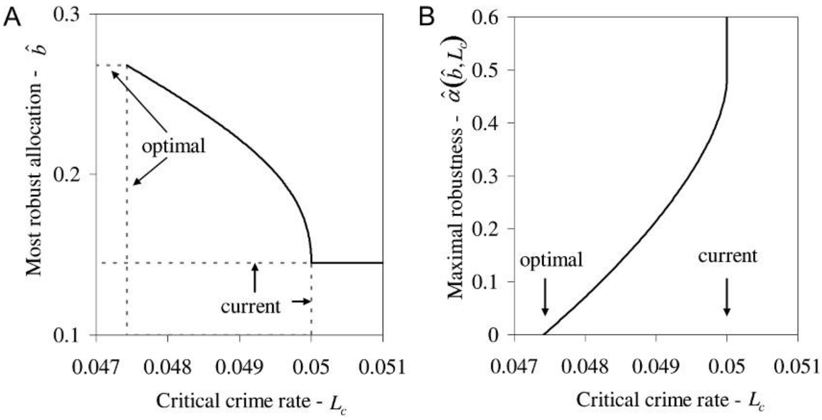

FIG. 2. (a) Displays the correspondence between the critical crime rate and the most robust allocation. In other words, for any critical crime rate, it shows the most robust allocation of policing resources. It can be proved that for most allocations, there can be only one critical crime rate for which the allocation will be most robust. (b) Illustrates the maximal robustness for any given critical crime rate. Note that as the critical crime rate increases (weaker requirement) the maximal robustness increases.

Another interesting result is that some allocations are 'robust dominated': for every critical crime rate, there is some other allocation with greater robustness. This is important since robust-dominated allocations should never be chosen. Sufficient conditions for an allocation to be robust dominated can be derived but will not be elaborated here.

The negative slope of Fig. 2a implies that as the critical crime rate decreases, the robust optimal allocation requires an increased allocation to young. This is not surprising since the young cohort has higher participation in crime. However, the large negative slope near the current crime rate implies that a robust-satisficing decision maker is unlikely to make a minor modification to the initial allocation. This is because small changes in the allocation are maximally robust only for negligible improvement in the crime rate. That is, there is a 'threshold effect' for the robust-satisficing decision maker: changes in the allocation are not robust optimal for a meaningful reduction in the crime rate until the change exceeds a particular threshold. This threshold is determined by the 'bend' in the curve in Fig. 2a and occurs around $b_y \approx 0.17$.

The slope of Fig. 2b may be viewed as the trade-off between critical crime rate (performance) and the maximal robustness. For instance, decreasing the critical crime rate by 0.001 entails a reduction in the maximal robustness by more than 0.1. That is, reducing the number of criminal incidents from 0.049 to 0.048 per person in a 14-month period, 'costs' a substantial reduction in the robustness to uncertainty from 0.21 to 0.08. Near the current allocation, the slope is very high (asymptotically infinite), implying that a small decrease in the critical crime rate has a great effect on the maximal robustness.

At any point of the curve of Fig. 2b, its slope equals the slope of the maximal robustness curve for that critical crime rate. The difference between maximal robustness at L_c and the robustness of the nominal optimal allocation at L_c is called the 'robustness premium' for the former allocation. A large robustness premium implies a strong preference for the robust optimal allocation over the nominal optimal allocation. The robustness premium is calculated as the difference between the curve and the tangent at the nominal optimum. The low curvature over most of Fig. 2b implies low robustness premium for maximal robustness over this range of L_c values. For instance, at $L_c = 0.049$, the robustness premium is $\Delta_{\hat{\alpha}} = 0.21 - 0.18 = 0.03$. The maximal robustness at $L_c = 0.049$ is 0.21, so the robustness premium is thus only about 15% of the maximal robustness. In other words, by choosing the nominal optimal allocation for the critical crime rate $L_c = 0.049$, we loose approximately 15% of the robustness. Conversely, the high curvature of Fig. 2b near the current allocation implies large robustness premium for the robust optimal allocation in that range. In summary, the policy implication of the curvature of Fig. 2b is that small reductions below the current crime rate have substantial robustness premium, while large reductions have small differences in robustness between the nominal and the robust optimal allocations.

This is different from the threshold effect mentioned earlier. The large robustness premium, for small changes in the current allocation, corresponds to very small improvement in the critical crime rate (this is the threshold effect). Large robustness premium by itself does not motivate the policy maker to change the allocation. The policy maker will require large robustness for acceptable (not negligible) reduction in crime.

What if the fraction of young change? This can happen gradually, as a result of a demographic change, or suddenly, by applying the our model to a specific sub-population (for instance, when considering the police enforcement in regions with different fraction of young drivers). Figure 3 illustrates the most robust allocation and the maximal robustness as a function of the fraction of young in the population.

The positive slope of Fig. 3a implies that as the fraction of young in the population increases, a robust-satisficing decision maker would increase the fraction of resources allocated to the young group. However, as Fig. 3b demonstrates, this does not mean that the robustness will also increase. The robustness has a turnaround effect—from some point, an increase in the fraction of young (the more responsive group) actually decreases the robustness. This is because, as the fraction of young increases, the most robust allocation tends to allocate more and more resources to the young group, thus moving further away from the observed allocation, which entails increased uncertainty in the responsiveness functions.

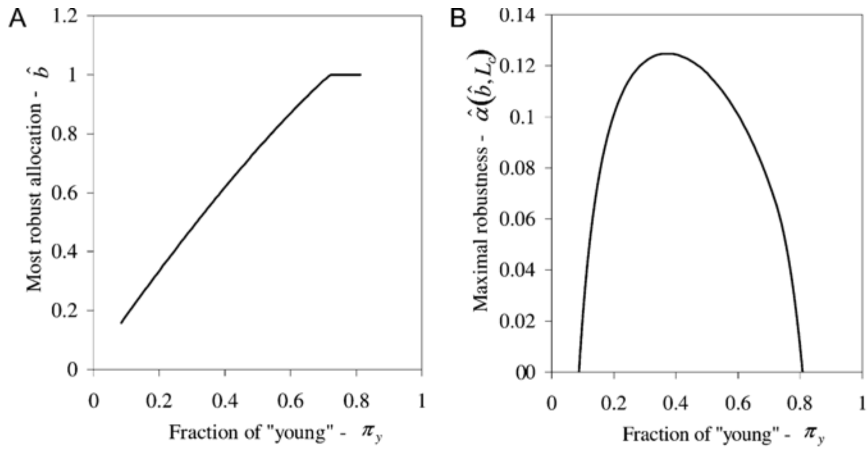

FIG. 3. (a) Displays the correspondence between the fraction of young in the general population and the most robust allocation. In other words, for any composition of the population, it shows the most robust allocation of policing resources. (b) Illustrates the maximal robustness for any given fraction of young within the general population. Both figures assume critical crime rate $L_c = 0.048$.

The slope of the two curves expresses the response of the allocation and the robustness to gradual changes in the composition of the population. For instance, the slope of Fig. 3a is only slightly less than unity. A change of 1% in the fraction of young within the population (relative to the current $\pi_y = 0.145$) will cause a change of only 0.9% in the robust optimal allocation. Thus, demographic changes are matched by similar changes in the robust optimal allocation. Similarly, a 1% change in the fraction of young results in a change of approximately 0.001 in the maximal robustness (about 1.5%). Thus, both the robust optimal allocation and the maximal robustness are will follow gradual changes in the fraction of the young drivers within the general population.

4. Dynamic analysis

In Section 3 we have considered a 'static' model of response to profiling: we considered only the crime rate as a function of policing resources allocated not the process in which these rates are achieved. However, when the allocation of policing resources change, the crime rate does not change instantaneously. It takes time for the population to accommodate to the new situation. During this time, the crime rate gradually converge to the crime rate described by the static model.

It is possible to apply info-gap analysis also to a 'dynamic' model, a model that describes the temporal change of the crime rate as a response to changes in the allocation. This kind of model also enables us to examine the case of different inter-temporal allocations. This can be done in many different ways. We will illustrate one simple but plausible approach, which is a direct generalization of our static analysis.

Let $C_i^{\mathrm{s}}(b_i)$ denote the static crime rate of group i given allocation of b_i policing resources. That is, after enough time, the crime rate of group i will converge to $C_i^{\mathrm{s}}(b_i)$.

We will denote the dynamic crime rate of group i at time t by $C_i^{\mathrm{d}}(b_i, t)$. In order to allow different inter-temporal allocations, b_i will be a vector, where $b_{i,1}, b_{i,2}, \ldots, b_{i,T}$ denotes the resources allocated to group i at times $t = 1, 2, \ldots, T$.

At time 0, we measured the crime rate C_i^0 of group i. These parameters are therefore certain. We will define $C_i^{\mathrm{d}}(b_i, 0) = C_i^0$. The dynamic crime rate at time $t > 0$ is calculated as follows:

$$C_i^{\mathrm{d}}(b_i, t) = (1 - \omega_i) \cdot C_i^{\mathrm{d}}(b_i, t-1) + \omega_i \cdot C_i^{\mathrm{s}}(b_{i,t}), \tag{3}$$

where $0 < \omega_i \leqslant 1$ is the convergence factor for group i. Intuitively, the dynamic crime rate is a weighted average of the past (the dynamic crime rate at the previous time step) and the future (the static crime rate). This is a standard adaptive learning model. Nonetheless, there is considerable uncertainty regarding the validity of this model in any specific application.

Our info-gap model, detailed in Appendix C, is an extension of the info-gap model described in Section 3: on top of the uncertainty in the static responsiveness to policing, C_i^{s}, there is also an uncertainty considering the the convergence factors, ω_i. Although we have a rough estimate, $\widehat{\omega}_i$, the actual value of ω_i may be anywhere between 0 and 1.

The loss function is the average crime rate:

$$L(b) = \sum_{t=1}^{T} \beta^t \sum_{i=1}^{n} \pi_i C_i^{\mathrm{d}}(b_i, t) \tag{4}$$

$$= \sum_{t=1}^{T} \beta^t \sum_{i=1}^{n} \pi_i \left(\sum_{\tau=1}^{t} (1-\omega_i)^{t-\tau} \omega_i C_i^{\mathrm{s}}(b_{i,\tau}) + (1-\omega_i)^t C_i^0 \right). \tag{5}$$

We discount the crime rate over time by inserting the term β^t, where $0 < \beta \leqslant 1$. This expresses the idea that future crime is currently less important than current crime.

4.1 *Robustness*

The robustness to uncertainty in the dynamic crime rate functions (more precisely, in the static crime rate functions and the convergence factors) is the greatest horizon of uncertainty up to which the loss does not exceed the critical value, L_c . Formally,

$$\widehat{\alpha}(b, L_c) = \max\left\{\alpha, \left(\max_{C^d,\omega\in\mathcal{U}(\alpha)} L(b)\right) \leqslant L_c\right\}. \tag{6}$$

We will exemplify robustness curves for the dynamic model for a two groups, two-steps problem. That is, we need to allocate policing resources to the young population and to the complementary group at time steps $t = 1, 2$. This problem is simple, yet gives good intuition as to the possible benefits of info-gap analysis.

We shall denote a temporal allocation by the matrix

$$b = \begin{pmatrix} b_{y,1} & b_{\bar{y},1} \\ b_{y,2} & b_{\bar{y},2} \end{pmatrix}, \tag{7}$$

where $b_{y,1}$ and $b_{\bar{y},1}$ denote the resource allocation at Time 1 and $b_{y,2}$ and $b_{\bar{y},2}$ denote the resource allocation at Time 2. Since the allocation refers to two time steps, it must hold that $b_{y,1} + b_{\bar{y},1} + b_{y,2} + b_{\bar{y},2} = 2$. This means that it is possible, for instance, to allocate resources in the present on the expense of future resources.

We will assume a discount factor of $\beta = 0.9$ and estimated convergence factors $\omega_y = \omega_{\bar{y}} = 0.75$. The estimated static crime response functions are as estimated in Appendix B.

Figure 4 illustrates the robustness curves of four temporal allocations described in Table 1: the current allocation maintains the current allocation (namely, the proportional allocation) throughout the following time steps; the optimal allocation brings $L(b)$ of (5) to minimum under the nominal assumptions; static is the optimal allocation for the static model; 'static1' and 'static2' preserves the ratio of allocated resources between b_y and $b_{\bar{y}}$ of allocation static but allocates more resources in time step $t = 1$ at the expense of time step $t = 2$ (static1) and vice versa (static2).

The vertical robustness curve for the current allocation indicates that, as is the case for the static model, this allocation is uncertainty free. The reason is that the static crime rate is certain and since the static crime rate is equal to the current crime rate, the uncertainty in ω_i is inconsequential. Therefore, the current allocation is the most robust allocation if the critical (discounted) crime rate L_c is 0.086 or higher.

Similar to the static model, the robustness curve of the optimal allocation is crossed by other robustness curves. This indicates that the optimal allocation is not the most robust for any critical crime rate. For instance, if the critical crime rate is higher than 0.083, then the static optimal (static) allocation is more robust.

As for temporal allocation, note the crossing of the robustness curves of the static optimal allocation and of static1. Recall that the static1 allocation is resulted from static by reallocating resources from time $t = 2$ to time $t = 1$. Since the future is discounted, there is some advantage in decreasing the crime rate now at the expense of a rise in the crime rate later in the future. Therefore, the nominal crime rate of static1 is lower than that of static. However, high horizon of uncertainty may result

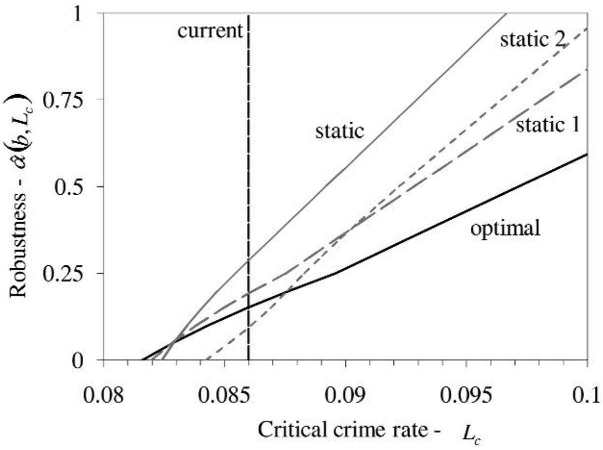

FIG. 4. Robustness curves of four allocations: the current allocation (current), the nominal optimal allocation (optimal) and three allocations based on the nominal optimal allocation for the static model: one that is time indifferent (static), one that emphasizes the first time step (static1) and one that emphasizes the second time step. The crossing of the curves indicates a change of preference: for instance, while the dynamic optimal allocation (optimal) is the most robust allocation (of the five presented) for critical crime rates no greater than 0.083, for most critical crime rates, the static optimal allocation (static) is more robust.

TABLE 1 *Temporal allocations*

Alternative		b_y	$b_{\bar{y}}$
Current	$t = 1$	0.145	0.855
	$t = 2$	0.145	0.855
Optimal	$t = 1$	0.283	1.015
	$t = 2$	0.248	0.454
Static	$t = 1$	0.268	0.732
	$t = 2$	0.268	0.732
Static1	$t = 1$	0.335	0.915
	$t = 2$	0.214	0.536
Static2	$t = 1$	0.214	0.536
	$t = 2$	0.335	0.915

allows high values of ω_i, which means that the discount of the future diminishes. In this case, the benefits of 'mortgaging the future' may be surpassed by the inefficiency (with respect to the undiscounted crime rate) of the allocation. The bottom line is that a mortgaging strategy, exemplified by the allocation static1, may lead to high nominal performance but low robustness to uncertainty.

The allocation static2 represents an opposite idea: investing more policing resources in the future ($t = 2$) at the expense of the present ($t = 1$). Since the crime rate is discounted, this allocation yields poorer nominal results in comparison with the static1 or even static. However, this allocation is less sensitive to uncertainty in ω_i, which may explain why the robustness curve for this allocation eventually crosses the robustness curve of static1. Although the robustness curves cross, this allocation is robust dominated (by the current and the optimal allocations for instance). This dominance conforms with the intuition that the temporal allocation should not contradict the discount of the crime rate.

5. Conclusion

The economic theory of crime views criminals as rational agents who adapt their behaviour in response to costs and benefits. This implies that involvement in criminal activity will respond with negative elasticity to changes in penalties or probabilities of apprehension. Since different groups respond differently, knowledge of the elasticities (or the responsiveness functions) would enable efficient allocation of enforcement resources. However, under a set budget, differential allocation of fixed total resources—profiling—can augment both the number of arrests and the total crime rate since non-profiled groups will increase their criminal activity. Specifically, profiling a minority can cause not only increased total arrests (mostly in the minority) but also increased total crime since the majority responds rationally to decreased enforcement by engaging in more crime.

We have focused on the problem of formulating a profiling strategy in light of the great uncertainty accompanying estimates of responsiveness to law enforcement. Since elastic response to profiling can result in increased total crime, the advocate of profiling must choose a strategy, which will not inadvertently result in this undesired outcome.

This paper has developed a robust-satisficing methodology for allocation of enforcement resources when the responsiveness functions are highly uncertain. We have used info-gap decision theory for satisficing (not minimizing) the total crime rate. We have demonstrated the trade-off between robustness to uncertainty on the one hand and reduction of total crime on the other hand. Attempting to minimize total crime has zero robustness to uncertainty in the responsiveness to policing. Since the responsiveness to policing is highly uncertain, low robustness is undesirable. Positive robustness is obtained only by aiming at a crime rate, which is larger than the estimated minimum. The robust-satisficing strategy chooses an allocation that guarantees an acceptable total crime rate (which usually will not be the estimated minimum) for the largest possible range of error in the estimated elasticities. The robustness analysis enables the decision maker to evaluate profiling options in terms of whether they promise adequate improvements in total crime at plausible levels of immunity to error in the responsiveness functions.

We have presented an empirical example based on measurement of the responsiveness to enforcement of traffic laws. We demonstrated a threshold effect: changes in the allocation are not robust optimal for a meaningful reduction in the crime rate until the change exceeds a particular threshold. We have also seen the effect of changing demographics on the robust-optimal profiling strategy. While the allocation changes approximately in parallel to the changing composition of the population, the robustness changes non-linearly, showing a maximum at an intermediate fraction of young drivers. Since it is the robustness premium that motivates adopting the robust-satisficing allocation, this implies that not all demographic changes should induce shifts in policy.

We exemplified info-gap analysis of a dynamic model, where the population does not response instantaneously to changes in the policing resource allocation. We have demonstrated that allocations that exploit a discount of the crime rate for allocating resources in the present at the expense of the future yields better nominal results but may be less robust to uncertainty.

We have not addressed the ethical aspect of profiling. However, we note that arguments for profiling, which are based on the utility of optimal profiling (rather than satisficing) based on best estimates of the responsiveness functions should be viewed skeptically. We have shown that optimal allocations have zero robustness to error and since responsiveness functions are highly uncertain, the purported benefits of optimal allocations are highly unreliable. If profiling can be justified on utilitarian grounds, such justification must rest on showing that desirable reduction of total crime can be obtained with adequate robustness to the main source of uncertainty (the responsiveness functions). That is, the strategy of robust satisficing is directly relevant to the ethical argument for (or against) profiling.

We have studied the profiling of two groups with uncertain responsiveness to policing and illustrated our results with estimated responsiveness to policing of running red lights. The extension of our results to multi-group profiling is straightforward.

Our dynamic analysis may be viewed as only 'semi-dynamic', as it focuses entirely on the dynamics of the population, neglecting the dynamics of the policing authority. An additional important extension would be to study the full dynamic interaction between enforcement and criminal activity in which each side learns about the other.

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Appendix A. Info-gap decision theory: a mathematical précis

Let $\widetilde{u}$ denote our best estimate of u, a parameter, vector, function and the like, which is used to estimate the loss $L(q, u)$ due to decision $q \in \mathcal{Q}$. An info-gap model is an unbounded family of nested sets, $\mathcal{U}(\alpha, \widetilde{u})$, of u values. As α gets larger, the sets become more inclusive. The info-gap model expresses the decision maker's beliefs about uncertain variation of u around $\widetilde{u}$.

Info-gap models obeys two axioms:

$$\text{Contraction}: \mathcal{U}(0, \widetilde{u}) = \{\widetilde{u}\}, \tag{A.1}$$

$$\text{Nesting}: \alpha < \alpha' \text{ implies } \mathcal{U}(\alpha, \widetilde{u}) \subseteq \mathcal{U}(\alpha', \widetilde{u}). \tag{A.2}$$

Contraction asserts that $\widetilde{u}$ is the only possibility in the absence of uncertainty, $\alpha = 0$. Nesting asserts that the sets become more inclusive as α gets larger.

Given a critical loss L_c, the 'robustness function' $\widehat{\alpha}(q, L_c)$ is the greatest level of uncertainty α which still guarantees a loss no greater than L_c:

$$\widehat{\alpha}(q, L_c) = \max \left\{ \alpha: \left(\max_{u \in \mathcal{U}(\alpha, \widetilde{u})} L(q, u) \right) \leqslant L_c \right\}. \tag{A.3}$$

Robust-satisficing decision making maximizes the robustness and satisfices the loss at the value L_c, without specifying a limit on the level of uncertainty:

$$\widehat{q} = \arg \max_{q \in \mathcal{Q}} \widehat{\alpha}(q, L_c). \tag{A.4}$$

Appendix B. Satisficing crime rate

Table B1 lists the crime rates of the two examined sub-groups, as measured by Bar-Ilan and Sacerdote (2004), before and after the increase to the fine. Based on these crime rates and the exponential model, depicted by (1), we were able to estimate the responsiveness parameters, thus estimating the approximated responsiveness functions of the different groups. Let b^0 denote the allocation of policing resources during the periods analysed by Bar-Ilan and Sacerdote (2004). As mentioned above, we assume that the allocation was fair, $b_i^0 = \pi_i$.

We may be fairly confident in the observed crime rate for b^0 at the time of measurement. However, it is reasonable to suppose that the uncertainty in the responsiveness function grows as the difference between the sampled resource allocation and the current resource allocation grows.

Let $\widetilde{C}$ be a vector of responsiveness functions, representing our best estimate of the responsiveness functions of the different groups. $\widetilde{C}_i$ will be the estimated model—i.e., the exponential model depicted in (1). We will refer to $\widetilde{C}$ as the nominal model and represent the uncertainty surrounding

TABLE B1 *Measured crime rates and responsiveness parameters*

Group	Crime rate			Responsiveness parameters	
	Before increase	After increase	Fraction	γ_i	δ_i
Age 17–30 (y)	0.123	0.056	0.145	1.31	1.57
Age 31+ ($\bar{y}$)	0.065	0.049	0.855	0.47	2.54

'Crime rate' is the mean number of tickets during the 14 quarter period before the fine increase and the 14 quarter period after the fine increase (Bar-Ilan and Sacerdote, 2004). 'Fraction' is the group's relative fraction within the general population of Israeli drivers based on a random sample of 1% of the Israeli drivers (Bar-Ilan and Sacerdote, 2004).

the actual responsiveness functions using the following info-gap model:

$$\mathcal{U}(\alpha, \widetilde{C}) = \left\{ C\colon C_i(b_i) \geqslant 0, \left| \frac{C_i(b_i) - \widetilde{C}_i(b_i)}{\widetilde{C}_i(b_i)} \right| \leqslant \alpha \left| \frac{b_i - b_i^0}{b_i^0} \right| \right\}, \quad \alpha \geqslant 0. \tag{B.1}$$

At any horizon of uncertainty α, the set $\mathcal{U}(\alpha, \widetilde{C})$ contains all non-negative responsiveness functions $C_i(b_i)$ which deviate from the nominal function by no more than $\alpha \left| \frac{b_i - b_i^0}{b_i^0} \right|$. Since α is unbounded, this is an unbounded family of nested sets of responsiveness functions.

The weight on the horizon of uncertainty (the absolute value term on the right-hand side of the inequality) means that for any given horizon of uncertainty, the uncertainty regarding the responsiveness grows as the allocation, b, deviates from the measured reference allocation, b^0.

Using (B.1), and recalling the definition of the robustness, one can readily show that for any allocation that distributes the policing resources between the groups y and $\bar{y}$, the robustness is

$$\begin{aligned} \widehat{\alpha}(b, L_{\mathrm{c}}) &= \max \left\{ \alpha\colon \left(\max_{C \in \mathcal{U}(\alpha, \widetilde{C})} \sum_{i \in \{y, \bar{y}\}} \pi_i C_i(b_i) \right) \leqslant L_{\mathrm{c}} \right\} \\ &= \frac{L_{\mathrm{c}} - \sum_{i \in \{y, \bar{y}\}} \pi_i \widetilde{C}_i(b_i)}{\sum_{i \in \{y, \bar{y}\}} \pi_i \widetilde{C}_i(b_i) \left| \frac{b_i - b_i^0}{b_i^0} \right|}. \end{aligned} \tag{B.2}$$

Appendix C. Satisficing crime rate: the dynamic model

First, note that

$$C_i^{\mathrm{d}}(b_i, 0) = C_i^0, \tag{C.3}$$

$$C_i^{\mathrm{d}}(b_i, 1) = (1 - \omega_i) \cdot C_i^0 + \omega_i \cdot C_i^{\mathrm{s}}(b_{i,1}), \tag{C.4}$$

$$C_i^{\mathrm{d}}(b_i, 2) = (1 - \omega_i)^2 \cdot C_i^0 + (1 - \omega_i)\omega_i \cdot C_i^{\mathrm{s}}(b_{i,1}) + \omega_i \cdot C_i^{\mathrm{s}}(b_{i,2}), \tag{C.5}$$

$$\vdots$$

$$C_i^{\mathrm{d}}(b_i, t) = \sum_{\tau=1}^{t} (1 - \omega_i)^{t-\tau} \omega_i C_i^{\mathrm{s}}(b_{i,\tau}) + (1 - \omega_i)^t C_i^0. \tag{C.6}$$

This explains (5), which in turn means that in order to satisfice the (discounted) crime rate under the dynamic model, we only need to refer to the uncertainty in the static responsiveness functions C_i^{s} and in the convergence factors ω_i.

Let $\widetilde{C}$ be a vector of responsiveness functions, representing our best estimate of the responsiveness functions of the different groups, and let $\widetilde{\omega}$ be a vector of our best estimates of the convergence vectors. We will refer to $\widetilde{C}$ and $\widetilde{\omega}$ as the nominal model and represent the uncertainty surrounding the actual functions and parameters using the following info-gap model:

$$\mathcal{U}(\alpha) = \left\{ C^{\mathrm{d}}\colon \forall\, i, \left| \frac{C_i^{\mathrm{s}}(b_{i,t}) - \widetilde{C}_i^{\mathrm{s}}(b_{i,t})}{\widetilde{C}_i^{\mathrm{s}}(b_{i,t})} \right| \leqslant \alpha \left| \frac{b_i - b_i^0}{b_i^0} \right|, \left| \frac{\omega_i - \widetilde{\omega}_i}{\widetilde{\omega}_i} \right| \leqslant \alpha, 0 < \omega_i \leqslant 1 \right\}. \tag{C.7}$$

Note that for any horizon of uncertainty α, the set of possible static responsiveness functions correspond to the possible responsiveness functions of the info-gap model of (B.1). In other words, the dynamic info-gap model is an expansion of the static info-gap model.

In contrast to the static case, there is no trivial closed-form expression for $\widehat{\alpha}(b, L_{\mathrm{c}})$ under the dynamic model.