Optimizing and Satisficing in Quantum Mechanics: An Info-Gap Approach

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Contents

Keywords: quantum indeterminacy, info-gap, Bell's theorem, EPR experiment, Schrödinger equation.

Abstract

Quantum mechanics introduces a probabilistic element into nature. We attempt to understand this non-probabilistically. We hypothesize that natural law is partially indeterminate. The basic idea is that the Lagrangian is indeterminate; it never takes a specific functional form. Consequently, a least-action principle is no longer applicable: there is no specific Lagrangian with respect to which the action can be stationary. Rather, we hypothesize that a specific concept of robustness to Lagrangian indeterminacy is stationary. We suggest that the Schrödinger equation can be derived by Feynman's 1948 path integral concept, except that now it is stationarity of the robustness, not of the action, which determines the quantum mechanical weight of a wave function. We hypothesize that the

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classical action depends on the path only through the robustness. Consequently, we obtain the same wave functions as in Feynman's method, but for a different reason, which sheds light on the issues of locality, realism and completeness. We discuss the absorption of polarized photons, Bell's theorem, and the EPR thought experiment. Our analysis is based on info-gap theory for representing non-probabilistic indeterminism in natural law. Several types of testable predictions are suggested.

1 Introduction

We hypothesize that quantum indeterminism originates from an indeterminism of natural law. This indeterminism undermines the concept of optimization as a strategy for deriving equations of motion. We replace the concept of optimization by the concept of satisficing as a means of understanding quantum indeterminism.

1.1 Optimizing and Satisficing

Optimization—finding stationary values of an objective function—is a fundamental concept in physics. The variational principles of mechanics are optimization problems, equilibrium in thermodynamics involves minimizing an energy function, and so on. The "optimization paradigm" asserts that laws which describe the behavior of a system can be derived by optimizing a physically meaningful objective function. The essence of each system is embodied in its objective function, whether it be an action integral or a Gibbs free energy or something else.

The optimization paradigm originated in physics and spread to natural and social sciences. For example, a large body of biological literature seeks to explain the foraging behavior of animals as guided by maximizing caloric intake. In social science, mathematical economists derive "equations of motion" of an economy from "first order conditions" which specify stationarity of economic utility functions.

But optimization requires that the objective function be determinate and accessible in some sense. This does not mean that the agents involved (projectiles, pigeons or pawn brokers for instance) need conscious knowledge of the objective function. Nonetheless, that function must be stable and discoverable—perhaps implicitly—in some relevant sense. Pigeons need not be conscious of their caloric intake, but measures or expressions of that intake must be accessible to them in some way, for instance by the evolutionary selection process against pigeons whose caloric intake is too low.

The need for stability and accessibility of the objective function led to a crisis of the optimization paradigm in social science. Simon [19, 20] studied what he called the "bounded rationality" of animal, human, and organizational decision makers. Bounded rationality is the behavioral consequence of limited access to information and understanding, and the limited ability to process that information. As a consequence, Simon claimed, organisms "satisfice". Etymologically, "to satisfice" is a variant on "to satisfy", but "satisfice" has come to have a tighter technical meaning in economics, psychology and decision theory. The Oxford English Dictionary [15] defines "satisfice" to mean "To decide on and pursue a course of action that will satisfy the minimum requirements necessary to achieve a particular goal."

1.2 Satisficing and Indeterminism

Satisficing—as an alternative to optimizing—is motivated when the objective function is partially indeterminate or unknown or of limited accessibility. In this paper we explore the satisficing paradigm for deriving physical theory—quantum mechanics in particular—and for developing a new understanding of quantum indeterminism.

The crux of our proposal is to hypothesize that natural law is not entirely determinate. This indeterminism underlies the stochastic character of quantum phenomena. It may be that this indeterminism is non-nomological—not governed by law—and thus outside the purview of science altogether.[1]

The basic idea is that the Lagrangian is indeterminate; it does not take a specific functional form for any given physical system. Consequently, a least-action principle is no longer applicable: there is no specific Lagrangian with respect to which the action can be stationary. Rather, we hypothesize that the robustness (to be defined) to Lagrangian indeterminacy is stationary. Quantum mechanically, paths are favored whose robustness functions are stationary with respect to path variation. Thus we will suggest that Feynman's 1948 derivation of the Schrödinger equation, based on the path integral concept, remains unchanged, except that now it is stationarity of the robustness, not of the action, which determines the quantum mechanical weight of a wave function. We hypothesize that the classical action depends on the path only through the robustness. Consequently, we obtain the same wave functions as in Feynman's method, but for a different reason, which sheds light on the issues of locality, realism and completeness.

Optimization is still central because stationarity of the robustness determines the equations of motion. But the robustness is not itself a property of the system, unlike the Lagrangian in both classical and quantum mechanical physics. The Lagrangian, which embodies the specific properties of the system, is indeterminate, and only the robustness to this property-indeterminism is optimized in deriving equations of motion.

We noted earlier that the optimization paradigm originated in physics and spread to natural and social sciences where it then faced a crisis arising from uncertainty. The present paper is an "echo" from social science back to physics; an adaptation of bounded rationality to quantum indeterminism.

Our discussion is based on info-gap theory [2, 12], which is a non-probabilistic methodology for modelling and managing uncertainty. Info-gap theory has been applied to design, decision and strategic planning under uncertainty in engineering, biological conservation, medicine, project management, economics, homeland security and other areas. One of the basic decision functions in info-gap theory is based on the idea of satisficing: meeting critical requirements or doing good enough, as distinct from optimizing an outcome. Info-gap robust decisions are ones which satisfice a requirement for the largest range of uncertain contingencies. This is called *robust-satisficing.* Robust-satisficing arises in this paper as a replacement for the idea of least action.

We introduce our approach by discussing the transmission of polarized photons in section 2. The core of the paper is sections 3 and 4. In section 3 we define the info-gap robustness function which underlies our approach, and we show how it is derived from the info-gap model for indeterminism. The info-gap model is the main new mathematical entity in the theory. In section 4 we state our formulation of the wave function in terms of the robustness-dependent action. In section 5 we present our alternative to Bell's concept of locality, and suggest that this alternative does not seem to contradict quantum mechanics. We discuss the EPR thought experiment in section 6. Our concluding discussion is in section 7. All proofs are in an appendix.

2 Polarized Photons

Dirac [7] discusses the interaction between polarized photons and the polarizing crystal tourmaline. If a beam of polarized light impinges at an angle α to the crystal axis, then a fraction $\sin^2\alpha$ will be transmitted and will be polarized perpendicular to the crystal axis upon exiting the crystal. If a single photon impinges, polarized at an angle α to the crystal axis, then it will either be completely

[1] This is reminiscent of the Bohr-Mottelson-Ulfbeck [5] idea of "genuine fortuitousness, … that the basic event, a click in a counter, comes without any cause and thus as a discontinuity in spacetime" (p.405). "The individual click with its discontinuity is entirely beyond law." (p.407). However, the proposal here is much more conventionally realistic in the metaphysical sense: there are particles "out there" which cause the clicks. What may be similar, though, is that the underlying undetermined element of natural law may be unfathomable. In this sense then I am pushing the Bohr-Mottelson-Ulfbeck boundary of science further back.

absorbed or completely transmitted; in the latter case it will be polarized perpendicular to the crystal axis. A fraction $\sin^2 \alpha$ of such photons, impinging independently, will be transmitted. After describing these observations Dirac writes [7, p.6]:

> Thus we may say that the photon has a probability $\sin^2 \alpha$ of passing through the tourmaline and appearing on the back side polarized perpendicular to the axis and a probability $\cos^2 \alpha$ of being absorbed. These values for the probabilities lead to the correct classical results for an incident beam containing a large number of photons.
>
> In this way we preserve the individuality of the photon in all cases. We are able to do this, however, only because we abandon the determinacy of the classical theory. The result of an experiment is not determined, as it would be according to classical ideas, by the conditions under the control of the experimenter. The most that can be predicted is a set of possible results, with a probability of occurrence for each.
>
> The foregoing discussion about the result of an experiment with a single obliquely polarized photon incident on a crystal of tourmaline answers all that can legitimately be asked about what happens to an obliquely polarized photon when it reaches the tourmaline. Questions about what decides whether the photon is to go through or not and how it changes its direction of polarization when it does go through cannot be investigated and should be regarded as outside the domain of science.

Two aspects of Dirac's interpretation need attention. First, he writes that "The result of an experiment is not determined ... by the conditions under the control of the experimenter." The lack of control is not the experimenter's but rather nature's lack. The classical conception of a law of nature is modified by quantum mechanics. When an event occurs it does so in one way and not any other (excluding multiple world interpretations). To this extent, some "law" or property inherent in the substances and circumstances may be said to be acting. However, if identical substances and circumstances at different instants or locations result in non-identical outcomes, then "natural law" as understood classically—immutable and universal—does not hold. There is some variability or indeterminism in those attributes which govern the course of events. What physicists have classically thought of as constant and fully specifiable laws are in fact to some extent indeterminate. It is not the experimenter's competence which is compromised but nature's competence to govern events.

This suggestion that natural law is indeterminate, as seen at the level of individual events, is fundamental to this paper. In section 3 we will formulate a sense in which the Lagrangian of a system is indeterminate. This indeterminism prevents us from applying a least-action principle. More generally, the optimization paradigm, as discussed in section 1.1, is no longer available as the basis for deriving natural laws such as Schrödinger's equation. Rather, we will derive the same equations by satisficing the action and optimizing the robustness to Lagrangian indeterminism. In a sense, then, the optimization paradigm is preserved, but the objective function is no longer a Lagrangian which represents well-defined properties of the system. The paradigm is robust-satisficing which involves an entirely new entity: an info-gap model of indeterminism.

Our second reservation concerns the domain of scientific investigation. Dirac asserts that "the main object of physical science ... is the formulation of laws governing phenomena" [7, p.10]. All events are, to some extent, singular and indeterminate occurrences, as illustrated by the polarized photon experiment. Dirac excludes the resolution of these singular indeterminacies from the domain of science. This exclusion is reasonable within the classical understanding of science as the identification and study of natural law representing determinate properties of physical systems. Dirac, however, asserts that nature is not entirely nomological. Dirac's position is that law is obeyed only on the average; that the idiosyncracies of individual events are not within the domain of science, the study of the laws of nature.

But if we take the view that science is the study of nature, and not only the discovery of immutable laws, then the exploration of the non-nomological aspects of natural phenomena, while not leading to the discovery of natural law, is legitimate science. More specifically, characterizing unsystematic and irregular elements of singular natural processes is most certainly a valid scientific pursuit, illustrated by quantum uncertainty principles of various sorts. The aim of this paper is to deepen our scientific understanding of natural indeterminism. Furthermore, in the process of this study we may in fact uncover new laws of nature which determine "whether the photon is to go through or not and how it changes its direction of polarization".

The exploration and characterization of indeterminacy in nature need not become a metaphysical pursuit, and can be reconciled with a thoroughly pragmatist or logical positivist philosophy of science. The scientific study of indeterminism is subject to the same criteria of warrant, e.g. Haack's foundherentism [11], which can be applied to all scientific theory.

The issue here is realism.[2] If science can say nothing about individual events, then is scientific theory consistent with solipsism—the denial by an individual of anything beyond the self? The "genuine fortuitousness" of Bohr, Mottelson and Ulfbeck [5] discussed earlier in footnote 1 on p.3 is a logical extension of Dirac's assertion that the life history of an individual photon "cannot be investigated and should be regarded as outside the domain of science." Perverse philosophical implications do not, by themselves, disprove an argument, but they should certainly alert us to the importance of studying the argument. It is claimed in this paper that idiosyncratic events can be understood—quantum mechanically—as real occurrences about which we can learn scientifically, although not in the conventional sense of identifying specific properties which determine specific outcomes.

3 Info-Gap Robustness

3.1 Formulation

First, some definitions. $x(t)$ is the classical trajectory of a particle. $L(x, \dot{x}, t)$ is a classical Lagrangian for the particle, which is a real scalar-valued function. $S(x, L) = \int_0^t L(x, \dot{x}, \tau)\, \mathrm{d}\tau$ is the action. $\mathcal{U}(\alpha, \widetilde{L})$, $\alpha \geq 0$, is an info-gap model for indeterminism of the Lagrangian, namely, an unbounded family of nested sets of Lagrangians. Each Lagrangian in these sets corresponds to different laws of nature for the same system.

Info-gap models obey two axioms [2]. *Contraction* states that, in the absence of uncertainty, only a single Lagrangian applies, so the uncertainty set is a singleton:

$$\mathcal{U}(0, \widetilde{L}) = \{\widetilde{L}\} \tag{1}$$

We refer to $\widetilde{L}$ as the nominal Lagrangian, which is the Lagrangian that a classical physicist would choose for the system under consideration. *Nesting* is the property that the sets become more inclusive as the horizon of indeterminism grows:

$$\alpha < \alpha' \quad \text{implies} \quad \mathcal{U}(\alpha, \widetilde{L}) \subseteq \mathcal{U}(\alpha', \widetilde{L}) \tag{2}$$

Mathematically, we define the robustness of a trajectory $x(t)$ as the greatest horizon of indeterminism, α, up to which the action is no greater than a critical value, S_c, for all possible Lagrangians at that horizon of indeterminism:

$$\widehat{\alpha}(x, S_\mathrm{c}) = \max \left\{ \alpha : \left(\max_{L \in \mathcal{U}(\alpha, \widetilde{L})} S(x, L) \right) \leq S_\mathrm{c} \right\} \tag{3}$$

[2]Realism has a plethora of meanings. The intention here is to two concepts, among others, discussed by Norsen [13]. I mean both metaphysical and perceptional realism. The former is the assertion that there is a world "out there", while the latter means that our senses provide us with at least partial access to that world.

The robustness, $\widehat{\alpha}(x)$, is the least upper bound of the set of α values which satisfice the action at its critical value. We define the robustness to equal zero if the set of α values in eq.(3) is empty.

The idea that we develop in this paper is that the Lagrangian is indeterminate. This means that the action cannot be minimized, since the action also is indeterminate. Our approach is based on the idea of *satisficing*: meeting a critical requirement, as distinct from optimizing a performance requirement. All paths have some level of robustness, defined in eq.(3). The **robust-satisficing path** is the one for which the **action is satisficed** at the value S_c and the **robustness is maximized**:

$$\widehat{x}(S_c) = \arg\max_x \widehat{\alpha}(x, S_c) \tag{4}$$

More generally, a robust-satisficing path makes the robustness stationary. For simplicity we will assume throughout our discussion that the robustness is stationary at its maximum.

3.2 Two Properties: Robustness and Zeroing

We prove and discuss several basic properties of info-gap robustness functions. For more details see [2]. We first need a definition.

Definition 1 *An action function* $S(x, L)$ expands continuously *on an info-gap model* $\mathcal{U}(\alpha)$ *if the function:*

$$f(\alpha) = \max_{L \in \mathcal{U}(\alpha, \widetilde{L})} S(x, L) \tag{5}$$

is a continuous function of α *for all* $\alpha \geq 0$.

Assuming the property of continuous expansion will simplify our proofs without unduely restricting the generality of our results.

Our first proposition states that robustness trades off against critical action: lower action is achieved only at lower robustness to Lagrangian indeterminism. This is an immediate result of the nesting of info-gap models of indeterminism, eq.(2).

Proposition 1 *Robustness trades off against the critical action.*
Given:
- *An info-gap model* $\mathcal{U}(\alpha)$ *and an action* $S(x, L)$ *which expands continuously on* $\mathcal{U}(\alpha)$.
- *The robustness function* $\widehat{\alpha}(x, S_c)$ *corresponding to* $\mathcal{U}(\alpha)$.
- *Two critical values,* S_{c1} *and* S_{c2}*, for which the robustnesses are positive and finite.*

Then *the robustness increases monotonically as the critical action increases:*

$$S_{c1} < S_{c2} \quad \textit{implies} \quad \widehat{\alpha}(x, S_{c1}) < \widehat{\alpha}(x, S_{c2}) \tag{6}$$

All proofs appear in an appendix, section 9

Definition 2 *An action* $S(x, L)$ *is* unsaturated *on an info-gap model* $\mathcal{U}(\alpha, \widetilde{L})$ *if, for all* $\alpha > 0$*:*

$$f(\alpha) > f(0) \tag{7}$$

We can understand the idea of unsaturation as follows. From the contraction property of info-gap models, eq.(1), we see that $f(0) = S(x, \widetilde{L})$. Unsaturation implies that even an infinitesimal expansion of the info-gap model to positive horizon of indeterminism α entails an increase in the maximum possible action at α.

The property of unsaturation immediately implies that the classical action has no robustness against indeterminism of the info-gap model, which we refer to as the property of **zeroing.** This is stated in the following proposition.

Proposition 2 *For any path, x, the robustness is zero when the critical action equals the nominal action $S(x,\widetilde{L})$.*

Given:

- *An info-gap model $\mathcal{U}(\alpha,\widetilde{L})$ and an action $S(x,L)$ which is unsaturated on $\mathcal{U}(\alpha,\widetilde{L})$.*
- *The robustness function $\widehat{\alpha}(x,S_{\rm c})$ corresponding to $\mathcal{U}(\alpha)$.*

Then *the robustness of the nominal action is zero:*

$$\widehat{\alpha}[x,S(x,\widetilde{L})]=0 \tag{8}$$

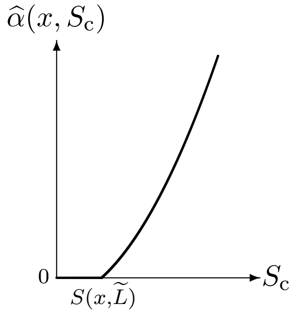

Figure 1: Robustness function.

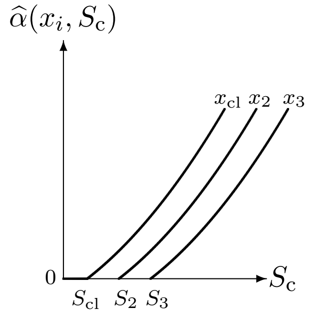

Figure 2: Robustness function.

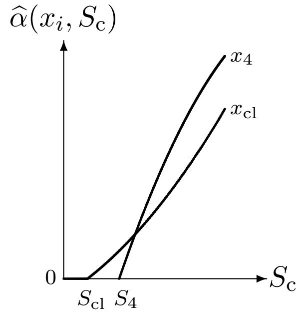

Figure 3: Robustness functions.

3.3 Discussion

We now discuss some implications of propositions 1 and 2. This discussion will take on an additional dimension after we relate the info-gap robustness to the Schrödinger equation in section 4.

Fig. 1 shows that robustness trades off against the critical action, illustrating proposition 1: lower (better) action, $S_{\rm c}$, entails lower (worse) robustness to indeterminism, $\widehat{\alpha}(x,S_{\rm c})$. The robustness vanishes when the critical action, $S_{\rm c}$, equals the nominal action, $S(x,\widetilde{L})$, as stated by proposition 2.

The robustness function, $\widehat{\alpha}(x,S_{\rm c})$, is evaluated for a particular path, $x(t)$. Fig. 2 illustrates robustness curves for three different paths, showing a common (though not universal) situation in which the robustness curves do not cross one another. The path labeled $x_{\rm cl}$ is the classical path: the one whose nominal action is minimal, denoted $S_{\rm cl}$. The other paths have larger nominal actions, S_2 and S_3. Proposition 2 asserts that the robustness is zero at the nominal action, which explains that the robustness curves of these three paths sprout off of the action-axis with $x_{\rm cl}$'s curve furthest to the left. In fig. 2 the classical path is more robust than path x_2 at all levels of critical action, and x_2 is likewise more robust than path x_3.

Fig. 3 shows a different situation: one in which the robustness curves of two paths cross one another, where one path is the classical path. The nominal action of path $x_{\rm cl}$ is less than the nominal action of path x_4 so $x_{\rm cl}$'s robustness curve sprouts off the action-axis to the left of x_4's curve. As a consequence $x_{\rm cl}$ is more robust than x_4 at low action. However, the robustness curves cross one another at a particular value of the critical action, call it $S_\times$. The robust-dominance is reversed and x_4 is more robust than $x_{\rm cl}$ at actions in excess of $S_\times$. More importantly, $x_{\rm cl}$ is not the robust-satisficing path at values of the critical action in excess of $S_\times$; some other path—perhaps x_4—maximizes the robustness.

4 Wave Functions and Robustness

4.1 Schrödinger's Equation and Stationarity of the Robustness

In this section we identify the relation between the info-gap robustness of a path, $\widehat{\alpha}(x,S_{\rm c})$, and the contribution of that path to the quantum mechanical wave function.

The 2nd axiom in Feynman's 1948 paper states:

> "*The paths contribute equally in magnitude [to the wave function], but the phase of the contribution is the classical action (in units of $\hbar$); i.e., the time integral of the Lagrangian taken along the path.*
>
> "That is to say, the contribution $\Phi(x)$ from a given path $x(t)$ is proportional to $\exp(i/\hbar)S[x(t)]$, where the action $S[x(t)] = \int L(\dot{x}(t), x(t))\,\mathrm{d}t$ is the time integral of the classical Lagrangian $L(\dot{x}, x)$ taken along the path in question." [9, p.371].

Consider an experiment in which a particle (classically speaking) moves from point A to point B. The contribution to the wave function (quantum mechanically speaking) for a specific path which goes through positions x_i and x_{i+1} at infinitesimally separated times t_i and t_{i+1} is:

$$\psi(x_i, x_{i+1}) = c \exp\left(\frac{i}{\hbar} S(x_i, x_{i+1}, \widetilde{L})\right) \tag{9}$$

where c is a constant for all paths and $S(x_i, x_{i+1}, \widetilde{L})$ is the action based on the nominal Lagrangian between times t_i and t_{i+1} for the path in question.

We accept Feynman's axiom but we add the **hypothesis** that the classical action, $S(x_i, x_{i+1}, \widetilde{L})$, depends on the path, x, only through the robustness function, $\widehat{\alpha}(x_i, x_{i+1})$. Thus eq.(9) becomes:

$$\psi(x_i, x_{i+1}) = c \exp\left(\frac{i}{\hbar} S[\widehat{\alpha}(x_i, x_{i+1})]\right) \tag{10}$$

We note that the action in eq.(10), $S[\widehat{\alpha}(x_i, x_{i+1})]$, still depends on the classical Lagrangian, as in eq.(9), because the robustness $\widehat{\alpha}(x_i, x_{i+1})$ depends on the classical Lagrangian through the info-gap model, $\mathcal{U}(\alpha, \widetilde{L})$. The wave function in eq.(10) will obey the Schrödinger equation if eq.(9) does. This depends first of all on the Lagrangian. In Feynman's original paper he assumed that "the action is the time integral of a quadratic function of the velocity." ([9] p.368). This actually does not hold universally ([16] p.163). Whether eq.(10) satisfies Schrödinger's equation also depends on the structure of the info-gap model for Lagrangian indeterminism. Specifically, the info-gap model must be such that the action depends on the path only through the robustness, which is our additional hypothesis. We will illustrate this in section 4.2.

The term $S[\widehat{\alpha}(x_i, x_{i+1})]$ in the exponent of eq.(10), which is the phase of the wave function, is very large compared to $\hbar$. If $\widehat{\alpha}(x_i, x_{i+1})$ is not stationary to variation of x then the phase varies rapidly for small changes in the path x, causing these paths to contribute very little to the probability density.

Suppose x is the robust-satisficing path, $\widehat{x}$ in eq.(4), at which the robustness is stationary. Now the phase is stationary to small variations around $\widehat{x}$, causing these paths to contribute to the probability density. From this we see that the robust-satisficing path, and paths very close by, will be dominant in the wave function for this particular robustness function. That is, let R be a set of paths whose wave function $\psi(R)$ is obtained by path-integrating $\psi(x_i, x_{i+1})$ over R (as in [9] sec. 5). If $\widehat{x}$ belongs to R then $\widehat{x}$ will dominate the integral, and if $\widehat{x}$ does not belong to R then the integral will be very small.

4.2 Simple Examples of the Robustness Function

We consider a simple example of info-gap robustness functions, based on an info-gap model for indeterminism of the Lagrangian. The adoption of an info-gap model is an assertion about the world, and we are in no position at the present to make such an assertion. This example is meant to illustrate how the wave function in eq.(10) can be assured to obey Schrödinger's equation, and to suggest a testable prediction.

Consider the following envelope-bound info-gap model for indeterminacy of the Lagrangian:

$$\mathcal{U}(\alpha, \widetilde{L}) = \left\{ L(x, \dot{x}, t) = \widetilde{L}(x, \dot{x}, t) + u(t) : \ |u(t)| \le \alpha w(x, t) \right\}, \quad \alpha \ge 0 \tag{11}$$

where $w(x, t)$ is a known non-negative real function which may depend explicitly on the trajectory, $x(t)$, and on time.

With this info-gap model, the robustness of trajectory segment x_i, x_{i+1} in the infinitesimal interval (t_i, t_{i+1}), defined in eq.(3), is:

$$\widehat{\alpha}(x_i, x_{i+1}, S_c) = \begin{cases} 0 & \text{if } S(x_i, x_{i+1}, \widetilde{L}) > S_c \\ \dfrac{S_c - S(x_i, x_{i+1}, \widetilde{L})}{\int_{t_i}^{t_{i+1}} w(x, \tau)\, d\tau} & \text{if } S(x_i, x_{i+1}, \widetilde{L}) \le S_c \end{cases} \tag{12}$$

This robustness function does not, in general, satisfy our hypothesis that the classical action depends on the path only through the robustness.

However, the hypothesis is satisfied if the nominal Lagrangian, $\widetilde{L}(x, \dot{x}, t)$, is non-negative[3] and the envelope function, $w(x, t)$, equals the nominal Lagrangian. Likewise the hypothesis is satisfied if the envelope does not depend on the path at all. In the first case, $w(x, t) = \widetilde{L}(x, \dot{x}, t)$, the positive part of the robustness function in eq.(12) becomes:

$$\widehat{\alpha}(x_i, x_{i+1}, S_c) = \frac{S_c}{S(x_i, x_{i+1}, \widetilde{L})} - 1 \tag{13}$$

In the second case, where the envelope function is $w(t)$ and does not depend on x, the positive part of the robustness function in eq.(12) becomes:

$$\widehat{\alpha}(x_i, x_{i+1}, S_c) = \frac{S_c - S(x_i, x_{i+1}, \widetilde{L})}{\int_{t_i}^{t_{i+1}} w(\tau)\, d\tau} \tag{14}$$

In both eq.(13) and (14), a path segment x_i, x_{i+1} has large robustness if its nominal action, $S(x_i, x_{i+1}, \widetilde{L})$, is small. Note that the robustness has a unique stationary point, occurring at the robust-satisficing path, if the action has a unique stationary point. Also, the robust-satisficing path does not depend on S_c.

Combining eq.(13) with eq.(9) results in the following expression for the contribution to the wave function of path segment x_i, x_{i+1}:

$$\psi(x_i, x_{i+1}) = c \exp\left(\frac{i}{\hbar} \frac{S_c}{1 + \widehat{\alpha}(x_i, x_{i+1}, S_c)} \right) \tag{15}$$

Combining eq.(14) with eq.(9) results in the following expression for the contribution to the wave function of path segment x_i, x_{i+1}:

$$\psi(x_i, x_{i+1}) = c \exp\left(\frac{i}{\hbar} \left[S_c - \widehat{\alpha}(x_i, x_{i+1}, S_c) \int_{t_i}^{t_{i+1}} w(\tau)\, d\tau \right] \right) \tag{16}$$

In both eq.(15) and (16), the robust-satisficing path, $\widehat{x}$, will dominate the wave function because the robustness is stationary at $\widehat{x}$.

Since the square of the wave function is observable, one can test the hypothesis that the info-gap model in eq.(11) actually describes natural indeterminism.

[3] For example, consider a point mass m falling freely from rest at initial position $y = 0$. The kinetic energy is $T = \frac{1}{2} m \dot{y}^2$ and potential energy is $V = -mgy$, so the Lagrangian is $L = T - V$ is non-negative.

4.3 Discussion

We now renew the discussion of the robustness curves in figs. 2 and 3 in light of the quantum mechanical interpretation of the robustness function based on eq.(10). We will assume, as before, that the robustness is stationary at its maximum.

Must Nature "choose" the critical action, S_c? Let us suppose that the robust-satisficing path, $\widehat{x}(S_c)$, defined in eq.(4) as the path which maximizes the robustness, is in fact independent of the critical value of the action, S_c. This means that there is a single path, $\widehat{x}(t)$, whose robustness curve (the plot of $\widehat{\alpha}(\widehat{x}, S_c)$ vs S_c) lies above the robustness curve of any other path. We now can conclude that $\widehat{x}$ equals the classical path, x_{cl}, since $\widehat{\alpha}(x_{cl}, S_c)$ is to the left—and thus above—the robustness curves of all other paths at least very near the horizontal axis. This is portrayed schematically in fig. 2.

Because the robustness is stationary for paths around $\widehat{x}$ one could imagine a great density of slightly different paths whose robustness curves lie slightly below the robustness curve for $\widehat{x}$ (which equals x_{cl} in fig. 2). The density of robustness curves would become lower as one moves further from $\widehat{\alpha}(\widehat{x})$, reflecting the greater rate of change of robustness with path variation. Finally, one can imagine that this variation of the density of paths is the same along any vertical line in fig. 2 defined by any value of S_c. In other words, the contribution to the wave function of different paths is the same for all values of S_c and thus nature never has to "choose" a value of S_c.

Are info-gap models empirically testable? The density of robustness curves depends on the info-gap model for Lagrangian indeterminism. But the density of robustness curves also determines the contribution of the corresponding paths to the wave function, whose square is an observable frequency of occurrence. One is thus able to test an info-gap model by its predictions of path frequencies.

Crossing robustness curves and the completeness of quantum mechanics. Now let us suppose that the robust-satisficing path, $\widehat{x}(S_c)$ defined in eq.(4), does in fact depend on the critical value of the action, S_c. That is, different paths maximize $\widehat{\alpha}(x, S_c)$ at different values of S_c. For instance, fig. 3 could represent a situation where $\widehat{x}(S_c) = x_{cl}$ for $S_c \le S_\times$ (where $S_\times$ is the value of S_c at which the robustness curves cross one another) and $\widehat{x}(S_c) = x_4$ for larger values of the critical action. Now the robustness, $\widehat{\alpha}(x, S_c)$, is stationary for paths around x_{cl} if $S_c < S_\times$, and it is stationary for paths around x_4 if $S_c > S_\times$. The contribution of paths to the wave function, according to eq.(10), will depend on the critical action.

What would this imply about the completeness of quantum mechanics? Suppose that quantum mechanics is a complete physical theory in the sense of Einstein, Podolsky and Rosen [8] to be discussed in section 6. This completeness of quantum mechanics would require that Schrödinger's equation, from which the wave function is derived, contains information about the critical value of the action. But Schrödinger's equation contains no such information, so either quantum mechanics is incomplete or robustness curves cannot cross one another. To put this differently, quantum mechanics is complete only if Nature never has to choose the critical action. Or again differently, quantum mechanics is complete only if the robust-satisficing path, $\widehat{x}(S_c)$, does not depend on the critical action S_c.

Crossing robustness curves and testing info-gap models. Whether or not the robustness curves of different paths cross one another depends on the info-gap model of indeterminism with which the robustness curves are constructed. If quantum mechanics is a complete physical theory then all info-gap models whose robustness curves cross one another are physically impossible.

5 Local Realism and Bell's Theorem

5.1 Bell's Theorem

Shimony [18] explains that "Bell's Theorem is the collective name for a family of results, all showing the impossibility of a Local Realistic interpretation of quantum mechanics." He continues that the Bell-type system "postulates an ensemble of pairs of systems". The two systems are denoted 1 and 2. *Realism* is the property that "Each pair of systems is characterized by a 'complete state' m which contains the entirety of the properties of the pair at the moment of generation."

Experiments on 1 are denoted a, a', etc., while experiments on 2 are denoted b, b', etc. Outcomes on 1 are denoted s, while outcomes on 2 are denoted t. A joint probability distribution $p_m(s,t|a,b)$, as well as its marginal distributions, $p_m^1(s|a)$ and $p_m^2(t|b)$, are assumed to exist (the superscripts emphasize which system is measured).

Bell Locality is the property of statistical independence, expressible in various equivalent ways, one of which is:

$$p_m(s,t|a,b) = p_m^1(s|a)p_m^2(t|b) \tag{17}$$

Now suppose there is a probability measure $\rho(\cdot)$ on the total state m, and that outcomes are constrained to the interval $[-1, 1]$. Then, as Shimony [18] explains, Bell shows that, if eq.(17) holds, then expectations of outcomes from experiments a, a', b and b', obey:

$$-2 \le \mathrm{E}_\rho(a,b) + \mathrm{E}_\rho(a,b') + \mathrm{E}_\rho(a',b) - \mathrm{E}_\rho(a',b') \le 2 \tag{18}$$

Bell then demonstrates a quantum mechanical system which violates this inequality. This prediction has been supported by numerous experimental studies, though not without loopholes as noted by Norsen [14] and as discussed extensively by Santos [17].

5.2 Info-Gap Locality

Aspect [1, p.867] asserts that "violation of Bell's inequalities implied that realism and locality are not simultaneously tenable." Similarly, Gröblacher *et al.* [10, p.871] write: "Bell's theorem proves that all hidden-variable theories based on the joint assumption of locality and realism are at variance with the predictions of quantum physics. Locality prohibits any influences between events in space-like separated regions, while realism claims that all measurement outcomes depend on pre-existing properties of objects that are independent of the measurement."

Bell locality, eq.(17), is the assertion of the *statistical* independence of (s,a) from (t,b), where s is the outcome of experiment a on system 1, while t is the outcome of experiment b on system 2. Statistical independence leads to Bell's inequality, eq.(18), which is violated by the predictions of quantum mechanics. However, statistical independence is not the only way to understand locality. Indeed, classical concepts of locality were not probabilistic. In this section we introduce a concept of info-gap locality, which does not imply Bell's inequality, and which does not seem to be contradicted by quantum mechanics.

The fundamental hypothesis in this paper is that the laws of nature are indeterminate, as manifested in indeterminism of the Lagrangian. This indeterminism is represented with an info-gap model, $\mathcal{U}(\alpha, \widetilde{L})$, which is non-probabilistic, as described in section 3. On the basis of this indeterminism we defined the robustness function in eq.(3). In particular, $\widehat{\alpha}(s|a,t,b)$ is the greatest horizon of indeterminism in the Lagrangian, up to which outcome s from experiment a on system 1 has small action, while outcome t resulted from experiment b on system 2. Likewise, $\widehat{\alpha}(s|a)$ is the greatest horizon of indeterminism in the Lagrangian, up to which outcome s from experiment a on system 1 has small action, regardless of what happens to system 2 or whether any experiment on 2 is performed at all. $\widehat{\alpha}(t|b,s,a)$ and $\widehat{\alpha}(t|b)$ are analogously defined.

Info-gap locality is defined by the following two assertions:

$$\widehat{\alpha}(s|a,t,b) = \widehat{\alpha}(s|a) \quad (19)$$

$$\widehat{\alpha}(t|b,s,a) = \widehat{\alpha}(t|b) \quad (20)$$

Info-gap locality is the property that the robustness (to natural indeterminism) of one system is independent of the robustness of another remote system. Like Bell locality, eq.(17), info-gap locality is formulated independent of quantum mechanics. Info-gap locality is an assertion about all realistic local theories, in which indeterminism is formulated in terms of info-gap models. Bell locality, similarly, deals with all realistic local theories which are analyzed probabilistically.

Info-gap locality is physically meaningful because the robustness function determines the phase of the wave function. Eqs.(19) and (20), together with eq.(10), imply that the contributions of (s,a) and (t,b) to the wave function of any system, including an entangled state, are:

$$\psi(s,a) = c\exp\left(\frac{i}{\hbar}S[\widehat{\alpha}(s,a)]\right) \quad (21)$$

$$\psi(t,b) = c\exp\left(\frac{i}{\hbar}S[\widehat{\alpha}(t,b)]\right) \quad (22)$$

These relations do not imply statistical independence of (s,a) and (t,b), and hence they do not imply Bell locality. Furthermore, they are consistent with an "entangled" wave function for systems 1 and 2 such as:

$$\Phi = \frac{1}{\sqrt{2}}\left[\psi(s,a)\psi(t,b) + \psi(s',a)\psi(t',b)\right] \quad (23)$$

A wave function of this sort underlies the violation of Bell's inequality.

The point is this. First, info-gap locality asserts that the degree of immunity (to indeterminism of the Lagrangian) of outcome s on system 1 is independent of what happens to the remote system 2, and vice versa. Info-gap locality is a non-probabilistic concept of locality which differs from Bell's probabilistic locality. Info-gap locality does not imply anything about likelihoods or expectations; specifically, Bell's inequality is not entailed. Second, info-gap locality is not contradicted by violation of Bell's inequality. Third, it would thus seem that info-gap locality is, for the time being, consistent with quantum mechanics.

6 EPR Thought Experiment

I find the EPR thought experiment deeply perplexing. In this section I will suggest one small contribution to a possible understanding.

The thought experiment. Consider the following form of the Einstein-Podolsky-Rosen (EPR) thought experiment [8], due to Bohm [3, 4]. A pair of electrons, whose total spin equals zero, is emitted from a source, one sent to position A and the other to position B. Each electron occupies a quantum state called a spin singlet, which is the superposition of two states, I and II. If electron A is in state I its spin points in the $+z$ direction and the B electron's spin points in the $-z$ direction. In state II the spin directions are reversed. If Alice, at position A, observed $+z$ spin then Bob at position B must observe $-z$ spin. If Alice observes $-z$ then Bob will observe $+z$. Thus Alice knows what Bob will observe immediately upon her observation.

Similarly, the electrons can be in states Ia and IIa defined analogously for the spins along the x axis. If Alice observes $+x$ spin then Bob will observe $-x$ spin, and if Alice observes $-x$ spin then Bob will observe $+x$ spin.

However, if Alice observes the z-axis spin and Bob measures the x-axis spin, then Bob has equal probability for observing $+x$ and $-x$, regardless of the result of Alice's observation. This is because x

and z spins are quantum mechanically incompatible: a Heisenberg uncertainty relation holds between them so they cannot simultaneously have precise values.

EPR's interpretation. EPR assumed that if the value of a physical quantity can be predicted with absolute certainty, before measurement, then that quantity is an *element of physical reality.* Next, they defined a *complete physical theory* as one which can account for all elements of physical reality. Finally, EPR adopted the *principle of locality* which states that physical processes at one location cannot have a superluminal effect (faster than the speed of light) upon the elements of physical reality at a different location.

EPR interpreted the thought experiment to mean that Alice's observation immediately determined an element of physical reality at Bob's location. If Alice observed $+z$ then Bob's z-spin is immediately determined, etc. EPR felt that this action at a distance violates the principle of locality. They were unwilling to relinquish this principle, so they felt that quantum mechanics is not a complete physical theory.

The info-gap interpretation. Let me focus on what one might mean by EPR's concept of an 'element of physical reality'. For this discussion it is important to recall what is meant here by 'realism', as mentioned in footnote 2 on p.5, namely both metaphysical and perceptional realism.

These concepts of realism imply that there are actually physical objects and they do actually have properties that we can perceive. Both classically and quantum mechanically we would say that these properties reflect laws of nature. For instance the hardness of a material, or the scattering of particles, reflect forces of interaction. These properties are the physical quantities which identify elements of physical reality.

However, the central idea explored in this paper is that the laws of nature are indeterminate. This must erode the concept of 'properties of objects' and hence also the concept of elements of physical reality. How can one accept an indeterminism of natural law without relinquishing metaphysical and perceptional realism?

The indeterminism does not mean that there are *no* laws of nature, or that there are *no* properties of objects, or that there are *no* elements of physical reality. It means that these laws—and properties—are in part incomplete. The laws of nature are not uniquely determined but rather fuzzy and indeterminate. This means that there are properties, but they too are—or at least might be—a bit fuzzy around the edges. When I say that an object is 'hard', the vagueness of this description is not only a property of human language. It is also a property of nature.[4] But the object is still 'there' (metaphysical realism) and I can still sense its hardness (perceptional realism).

And when I say 'this electron has spin in the $+z$ direction' there really is something there and it is "spinning" in some (non-macroscopic) sense. Now the perception is crisp though the laws are still indeterminate.

So what about EPR's concept of an 'element of physical reality'? The prediction of a perception may be precise, though it need not be. In both cases the laws are indeterminate. Prediction relates to perceptions, and ex post perceptions may be crisp (like '$+z$ spin' or 'polarized photon was transmitted') without requiring that the laws of nature are determinate. Elements of physical reality are there and can be perceived at least in part, though their properties reflect laws of nature which are not entirely determinate.

A modified version of Plato's cave metaphor is helpful. Fuzzy images on the wall may arise from fuzzy fires behind us. But even crisp perceptional reports (e.g., 'shadows on the wall exist') can arise from fuzzy fires.

[4]See [2, section 13.2] for a closely related discussion of ontological and epistemic uncertainty.

7 Discussion

Optimizing, satisficing and indeterminism. This paper is an attempt to understand the origin of quantum indeterminism. It is proposed that quantum indeterminism results from an indeterminism of natural law. The approach is to replace the conventional—both classical and quantum mechanical—concept of optimization by the idea of satisficing. This motivates the info-gap concept of robustness to Lagrangian indeterminism. We have shown how quantum mechanics can be derived in a Feynman-like manner by hypothesizing that the action depends on the path only through the robustness.

Realism. We have replaced Bell's "complete state m" with a Lagrangian, L. Bell's m and our L each vary between different pairs of states in the ensemble of possible events (or worlds). In Bell's case, this variation is probabilistic and he postulates a normalized probability density function (pdf), $\rho(m)$, with finite moments. In our case this variation is non-probabilistic and characterized by an info-gap model $\mathcal{U}(\alpha, \widetilde{L})$. There may, or may not, be a normalized pdf on the space of Lagrangians. Or, even if a pdf exists, it may or may not have finite moments. Realism, in our formulation, is the assertion that events are real and we can perceive them, and that each event is described quantum mechanically by the robustness to Lagrangian indeterminism as expressed by an info-gap model. Events are idiosyncratic and indeterminate because of the indeterminism of the Lagrangian.

Bell's conclusion that *probabilistic* local realism is inconsistent with quantum mechanics still holds. However, in our formulation, the laws of nature are indeterminate, and probability is not fundamental in formulating this indeterminacy. The wave function is formulated Feynman-like, but with the added hypothesis that the classical action depends on the path only through the robustness.

Hidden variables. Our approach bears some superficial similarity to "hidden variable" theories: the complete state m and the laws of nature which underlie the Lagrangian L are both "hidden" from quantum mechanics. Obviously laws of nature differ from the complete state of a system. The laws of nature are a holistic (though indeterminate) specification of the world, while the state refers to a part of that world. More importantly, our formulation of the indeterminism of the Lagrangian is not probabilistic. We do not presume that a probability function, or its moments, exist for L, and yet we still obtain ordinary quantum indeterminacy.

Completeness. Stapp [21] defines what d'Espagnat [6, p.49] calls the 'weak completeness' of quantum mechanics as the assumption that "no theoretical construction can yield experimentally verifiable predictions about atomic phenomena that cannot be extracted from a quantum theoretical description." In our construction this means that quantum mechanics is weakly complete if all "experimentally verifiable predictions about" Lagrangian indeterminism and its representation by an info-gap model can be obtained quantum mechanically. Such an assertion is contingent, and could conceivably be falsified, just like Stapp's assertion about the (ordinary) weak completeness of quantum mechanics. This is not to say that we cannot scientifically study the info-gap model for uncertainty in the Lagrangian. (Some testable predictions regarding info-gap models are suggested in section 4.3.)

Locality. In our formulation, locality is quantified with the info-gap robustness function whose physical importance arises through the phase of the wave function, as in eq.(10), and the hypothesis in section 4.1 that the action depends on the path only through the robustness. Info-gap locality is the assertion that the robustnesses of separated systems are independent functions, eqs.(19) and (20). Thus locality is the assertion that phase is, at least partially, a local phenomenon. Info-gap locality still allows quantum entanglement, and it does not imply Bell locality, which is a probabilistic property. It seems that info-gap locality is consistent with quantum mechanics. An open question is whether info-gap locality, as we have defined it, is the strongest concept of locality which is consistent with quantum mechanics.

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9 Appendix: Proofs

Proof of proposition 1. We will use the definition of $f(\alpha)$ in eq.(5). For notational compactness define $\widehat{\alpha}_i = \widehat{\alpha}(x, S_{ci})$.

From the definition of robustness, eq.(3), and since $\widehat{\alpha}_i$ is positive:

$$f(\widehat{\alpha}_i) \le S_{ci} \tag{24}$$

Suppose that:

$$f(\widehat{\alpha}_i) < S_{ci} \tag{25}$$

Then, since $S(x, L)$ expands continuously on $\mathcal{U}(\alpha)$, there exists $\epsilon > 0$ such that $f(\alpha_i + \epsilon) < S_{ci}$. Hence, again from the definition of robustness:

$$\widehat{\alpha}_i + \epsilon \le \widehat{\alpha}(x, S_{ci}) \tag{26}$$

which is a contradiction. Thus eq.(25) is false and eq.(24) implies:

$$f(\widehat{\alpha}_i) = S_{ci} \tag{27}$$

By supposition, $S_{c1} < S_{c2}$. Thus eq.(27) implies:

$$f(\widehat{\alpha}_1) < f(\widehat{\alpha}_2) \tag{28}$$

The nesting axiom, eq.(2), implies that $f(\alpha)$, defined in eq.(5), cannot increase as α decreases. Hence eq.(28) implies that $\widehat{\alpha}_1 < \widehat{\alpha}_2$ which is the desired result. ▮

Proof of proposition 2. The robustness is non-negative by definition. By the contraction property of info-gap models, eq.(1), we see that $f(0) = S(x, \widetilde{L})$. By the unsaturation of $S(x, L)$ we see that $f(\alpha) > S(x, \widetilde{L})$ for all $\alpha > 0$. Thus $\widehat{\alpha}[x, S(x, \widetilde{L})]$ cannot be strictly positive, which proves eq.(8). ▮