Appearing in
*Susan Haack, A Lady of Distinctions: The Philosopher Responds to Her Critics,*
edited by Cornelis de Waal, Prometheus Books

# Peirce, Haack and Info-gaps

Yakov Ben-Haim
Yitzhak Moda'i Chair in Technology and Economics
Technion — Israel Institute of Technology
Haifa 32000 Israel
yakov@technion.ac.il
http://www.technion.ac.il/yakov

Surprise and change are the way of the world. Philosophers have known this at least since Thales, and practical men knew it long before. Variety and the continual flux of one thing into another is, for Peirce, a central notion. A very similar conception underlies the information-gap theory of uncertainty and its application to decisions with severely deficient understanding which I have argued for earlier [1–6]. For Haack, whose treatment of warrant is strongly non-probabilistic, info-gap theory is a natural context. The connection between Peirce, Haack and info-gap theory is explored in this paper.

I will interpret a few ideas appearing in the work of Peirce and Haack — synechism, fallibilism, evolution and warrant — from the perspective of the info-gap theory of decision under uncertainty. Neither Peirce nor Haack had any familiarity with info-gap theory when they wrote the lines which I will analyze. We will nonetheless discover clear conceptual continuity between Peirce, Haack and central info-gap ideas.

A truism in many areas of quantitative analysis is that the best quantitative model, the state-of-the-art, will almost inevitably soon be out of date. Info-gap decision theory is a methodology for using quantitative models to make strategic design or planning decisions when the best available models are highly suspect of being seriously wrong. An info-gap is the disparity between what *is known* and what *should be known* in order to make a responsible decision. Info-gaps arise in engineering design with new materials whose properties are incompletely understood. Severe info-gaps are common in innovative integration of diverse technologies. But even so prosaic and hoary a pursuit as the design of buildings against earthquakes is faced with enormous disparity between the best quantitative characterizations of seismic loads and the real site- and event-specific complexity of soil-structure interactions. Outside of engineering, info-gaps characterize epistemic uncertainty in industrial management, economic behavior, medical decision-making, conservation biology, and virtually every other field in which state-of-the-art models are used to support decisions.

Info-gap theory attempts to model and manage uncertainty and indeterminism in their least structured and most pernicious form. An info-gap model of uncertainty is non-probabilistic and is based on the idea that unknown possibilities run fluidly one into another, that patterns and connections occur in endless variety, that the horizon of potential realizations is boundless, and that likelihoods and probabilities are advanced and sophisticated judgments far removed from the underlying phenomenon of uncertainty.

The most fundamental theorem of info-gap theory asserts that a system's robustness to uncertainty decreases as the functional requirements for that system become more demanding. A highly ambitious design will be vulnerable to info-gaps; designs based on optimization of performance are highly unreliable when the design-base models are highly uncertain. The primary design strategy of info-gap theory is to *satisfice the performance* — meet basic or minimal functional requirements — and to *maximize the robustness* to uncertainty in the underlying models. This is very different from the usual strategy of using the best model to maximize the performance.

[0]\papers\haack_vol\phig03.tex 21.10.2004.

Peirce would have liked the info-gap conception of uncertainty and its implications for explanation. Regarding Haack I have to be more circumspect, but we will find the seeds of info-gap ideas in her work. This will indicate both an interesting connection between Peirce and Haack and some new philosophical problems calling for attention.

Uncertainty, for Peirce, is a cardinal attribute of reality. One grasps what Peirce means by "understanding reality" only by grasping Peirce's notion of uncertainty. This notion is an info-gap conception and is as distant from probability as East is from West. I will first present Peirce's 'synechism' construed with info-gap theory. I will then discuss fallibilism, evolution and info-gap indeterminism. Finally I will discuss warrant.

## 1 Synechism: Naturalists and Info-gap Models of Uncertainty

Fortunately 'synechism' can be described in clear and graphic terms, which Peirce does skillfully:

> When a naturalist wishes to study a species, he collects a considerable number of specimens more or less similar. In contemplating them, he observes certain ones which are more or less alike in some particular respect. They all have, for instance, a certain S-shaped marking. He observes that they are not *precisely* alike, in this respect; the S has not precisely the same shape, but the differences are such as to lead him to believe that forms could be found intermediate between any two of those he possesses. He, now, finds other forms apparently quite dissimilar — say a marking in the form of a C — and the question is, whether he can find intermediate ones which will connect these latter with the others. This he often succeeds in doing in cases where it would at first be thought impossible; whereas, he sometimes finds those which differ, at first glance, much less, to be separated in Nature by the non-occurrence of intermediaries. [18] or [24, p.62]

One wonders if D'Arcy Thompson read this passage before engaging in his rubber-sheet transformations of projections of horse skulls and fish profiles to test the phylogeny of fossil sequences [29, chap. IX]. Thompson thought that he could detect false phylogeny by occurrence of morphological discontinuity, and conversely, that morphological continuity supported phylogenetic continuity.

Peirce employs the concept of 'occurrence of intermediaries' to describe spontaneous variation around a norm. He generalizes the gradation of one form into another by "the idea of continuity, or the passage from one form to another by insensible degrees" [18] or [24, p.63]. By continuity in this context Peirce means a generic 'connectedness' or 'between-ness' which characterizes a class of similar but varying entities. But I doubt that Peirce meant to gerrymander reality, to suggest that D'Arcy Thompson could establish phylogeny by exotic though continuous morphological contortions. The vertex at the bottom of a caligraphic "$\mathcal{V}$" is 'between' the tips of the "horns", since it is 'continuously connected' to them via the sides of the "$\mathcal{V}$". This cannot be what Peirce intends, so neither connectedness nor continuity is quite the right mathematical metaphor.

Convexity, rather than continuity, is a better though still imperfect mathematical model for similar but varying entities. A set is convex if it entirely contains the line segments joining any two points in the set. That is, a set is convex if it contains all linear intermediaries between every two points in the set. Convexity, and not simply continuity, is closer to characterizing the clustering tendency of diverse and variable but related phenomena.

We can now explore the connection between Peirce's principle of continuity and the idea of info-gap uncertainty. An info-gap model is an unbounded family of nested sets. The elements of the sets represent events or instances, the naturalist's specimens for example. But the sets do not contain only those specimens which have actually been collected. Each set in an info-gap family of sets contains an infinity of unactualized possibilities. The sets of an info-gap model are usually (though not invariably) convex, representing the 'between-ness' which characterizes the familial connection among the members of the set. For further examples of convex info-gap models see [1].

An info-gap model represents that facet of a phenomenon about which our understanding is incomplete or about which the phenomenon itself is indeterminate. Info-gaps can be either epistemic or ontological. Each set of an info-gap family corresponds to a given horizon of uncertainty. But since the horizon of uncertain variability is either unknown or indeterminate, the info-gap model is an unbounded family of sets. The sets are *nested,* one within another, and the level of nesting of each set is specified by its horizon of uncertainty. The sets become more inclusive as the horizon of uncertainty increases. Each set contains all possibilities at a given horizon of uncertainty, and greater uncertainty adds but cannot remove possibilities.

An info-gap model provides a mathematical representation of what Peirce refers to as spontaneous variability:

> It is evident ... that we can have no reason to think that every phenomenon in all its minutest details is precisely determined by law. That there is an arbitrary element in the universe we see, — namely, its variety. This variety must be attributed to spontaneity in some form. [19] or [24, p.175]

The variability to which Peirce refers is not expressible by highly structured concepts such as probability. Peirce's 'spontaneous variability' is an amorphous and non-nomological but nonetheless very real range of potential realizations. Peirce's principle of continuity makes only the weakest claims about the limits of variability or the relations among the variates. In this respect Peirce's uncertainty has very little in common with highly structured theories such as probability theory or more recent mathematical models of uncertainty [8] which all represent uncertainty in terms of real-valued distribution functions of probability, possibility, necessity, plausibility, etc.

To be more specific, the Kolmogorov axioms of probability [14] impose a specific and informationally intensive structure on the uncertainty model. A probability distribution makes specific assertions about exceedingly rare events, and the Kolmogorov axioms impose strong constraints (such as additivity, normalization, etc.) on the allowed probability distributions. This is utterly different from the much weaker axioms of info-gap models which entail no measure functions at all. Kolmogorov was perhaps aware of the strong ontological content of probability when it is construed as a model of the world, for he insisted on putting probability theory in its "natural place, among the general notions of modern mathematics", and he makes no more than a passing reference to the "concrete physical problems" from which probability theory arose [14, p.v]. An axiomatic comparison of info-gap models with probability is found in [3].

The idea of an info-gap as either lack of knowledge or inherent indeterminism corresponds well to Peirce's conception of uncertainty. Peirce's 'continuity' is represented by set-nesting, while no further constraints on relations among variates are imposed. As we will see, the decision theory which derives from info-gap models of uncertainty shows continuing points of contact with Peirce's thought.

## 2 Fallibilism

We have seen that info-gap models provide a mathematical representation of Peirce's conception of continuous and uncertain variability. Peirce's principle of continuity helps to understand his idea of fallibilism. Synechism, writes Peirce, is

> that tendency in philosophical thought which insists upon the idea of continuity as of prime importance in philosophy and, in particular, upon the necessity of hypotheses involving true continuity.
>
> A true continuum is something whose possibilities of determination no multitude of individuals can exhaust. ...
>
> In like manner, it is not a hypothesis fit to be entertained that any given law is absolutely accurate. [21]

Elsewhere Peirce writes:

> The principle of continuity is the idea of fallibilism objectified. For fallibilism is the doctrine that our knowledge is never absolute but always swims, as it were, in a continuum of uncertainty and of indeterminacy. Now the doctrine of continuity is that *all things* so swim in continua. [20] or [23, p.356]

I will now explain how info-gap decision theory provides a quantification of these ideas. By using the info-gap robustness function (to be defined) to measure warrant we have a mathematical model of Peirce's conception of fallibilism as it derives from his principle of continuity. This is of prime importance in bringing Peirce's principles into practice, as well as for understanding the epistemological status of a very practical decision theory.

The assertion that "all things" swim "in a continuum of uncertainty and of indeterminacy" is a pure info-gap concept! Add to that the contention that no law is "absolutely accurate" and we come to the concept of an info-gap robust test of truth [4, section 12.4.1], which I now discuss.

Consider an hypothesis $H$ which may be an assertion of fact like 'The snark was a Boojum' or a generalization like Ohm's law that 'The electric current in resistive material is proportional to the voltage'. Or the hypothesis may be a prediction such as 'If the central bank lowers the interest rate then unemployment will decrease'. Or $H$ may be a grand theory of the universe (or at least of some part of it) such as Darwin's evolutionary doctrine or Einstein's general relativity. The hypothesis passes an info-gap robust test of truth to the extent that its robustness to info-gaps is large.

As a preliminary definition, a proposition's robustness to info-gaps is the greatest level of info-gap uncertainty at which specified minimal requirements are satisfied by the proposition. This expresses, in embryonic form, the basic idea of satisficing on an info-gap family of nested sets of uncertain realizations. The robustness of a proposition can be interpreted as a measure of warrant for an hypothesis $H$ based on evidence $E^\star$ in the following way.

We understand "evidence" in a very broad sense: measurements, models, understanding of the relevant processes, and so forth. The evidence $E^\star$, however, is uncertain, it swims in Peirce's continuum: the measurements could have come out differently, the models could be erroneous, our understanding of the factors governing the processes could be faulty or incomplete, the processes themselves may be indeterminate, etc. These uncertainties are encoded in an info-gap model of uncertainty. At any given horizon of uncertainty there is an infinite continuum of different possible bundles of evidence $E$ which could have been realized. In short, we have an info-gap model of uncertain evidence in which $E^\star$ is the center point around which the sets of the info-gap model are nested.

We can now state in approximate verbal form the robustness of the assertion '$E^\star$ implies $H$':

> The robustness of the assertion '$E^\star$ implies $H$' is measured as the greatest horizon of uncertainty (in the info-gap model) up to which every bundle of evidence $E$ adequately supports[1] $H$.

The robustness of a proposition is the greatest deviation of possible evidence, $E$, from the specific evidence in hand, $E^\star$, which still adequately supports the hypothesis we wish to test. The robustness is the answer to the question: 'how wrong can the evidence $E^\star$ be without jeopardizing the adequacy of support for $H$?'. The demand for no more than adequate (rather than overwhelming) support for the hypothesis is a satisficing rather than an optimizing requirement. What the robust-satisficing inquirer does, in the search for strongly warranted assertions, is to seek hypotheses for which adequate support is maximally robust. (See [4], especially section 3.1, for further intuitive discussion of robustness and its mathematical formulation.)

Now suppose that hypothesis $H$ has adequate support from the evidence in hand, $E^\star$, (recall that we construe 'evidence' broadly) and has maximal robustness (to info-gaps in $E^\star$) from among competing hypotheses. This means that $H$ wins the info-gap robust test. Nonetheless one cannot assert that $H$ is "absolutely accurate". Some competing hypotheses may have adequate support with only slightly less robustness, or may have more than adequate support with substantially less robustness. Under-determination and lack of conclusive warrant are fairly unavoidable in inference under info-gap uncertainty. This is a direct result of the structure of the info-gap model, which quantifies Peirce's principle of continuity: universal variability and indetermination.

To take a case in point let hypothesis $H$ be Ohm's law mentioned before: 'The electric current in resistive material is proportional to the voltage'. This is an instance of the huge class of linear relations which play pivotal roles in every domain of quantitative analysis.[2] An hypothesis such as Ohm's law is tested by assessing its fidelity to data. The hypothesis is warranted in an info-gap robust test by adequately high fidelity together with large robustness against neglected non-linear terms (and perhaps other factors such as noise in the data). Large robustness means that ignoring large non-linear terms does not jeopardize the fidelity between model and data.[3] When $H$ passes the info-gap robust test with flying colors this does not mean that there is no non-linearity, nor does it mean that the linear law, as a sharp and anomalous point in the space of possible laws, is correct. Like Peirce, info-gap theory claims that an exclusive and unitary hypothesis is not worth its salt. What info-gap robust warrant implies is that the law could be very wrong and yet still true to the evidence; that we have no ground for rejecting the linear law on the basis of available evidence; that as far as we know there is no absolute law, the phenomenon may well be indeterminate, but Ohm's assertion captures the evidence. (For detailed examples see [4] and [6].)

The non-uniqueness of warranted assertions derives directly from the continuous uncertain variability which is represented by an info-gap model of uncertainty. Peirce's synechism — his principle of continuous variability — leads directly to the non-sharpness of warranted hypotheses, to the meaninglessness of an assertion of definitive warrant in an inference from evidence. Furthermore, and very importantly, Peirce's conclusion of the non-absoluteness of inductive law arises in info-gap decision theory without supposing the detailed and specific structure of uncertainty which is inherent in the theory of probability.

A similar sort of non-absoluteness can be culled from statistical decision theory, and is inherent in Neyman-Pearson hypothesis testing for instance. However, an important distinction between probabilistic and info-gap indeterminism is that the latter makes weaker and more Peirce-like assertions about the underlying uncertain variability, as we mentioned at the end of section 1.

## 3 Evolution

Peirce writes:

> But fallibilism cannot be appreciated in anything like its true significancy until evolution has been considered.
>
> Evolution means nothing but *growth* in the widest sense of that word. ... And what is growth? Not mere increase. ... [Rather] *diversification.* ... And yet mechanical law, which the scientific infallibilist tells us is the only agency of nature, mechanical law can never produce diversification. ... [M]echanical law out of antecedents can only produce like consequents. It is the very idea of law. So if observed facts point to real growth, they point to another agency, to spontaneity for which infallibilism provides no pigeon-hole. [20] or [23, p.357]

Peirce's understanding of 'mechanical law' may perhaps require revision in light of the probabilistic laws of quantum mechanics and the chaotic behavior of some non-linear systems. But this would not alter Peirce's idea of 'growth' and 'diversification' since Peirce intends something very different from laws of nature which dictate knowable probability distributions of possible outcomes. Peirce is thinking about the indeterminate and unknowable:

> We look back toward a point in the infinitely distant past when there was no law but mere indeterminacy; we look forward to a point in the infinitely distant future when there will be no indeterminacy or chance but a complete reign of law. [20] or [23, p.358]

I will propose a mechanism for Peircean indeterminacy in evolution, and for its eventual disappearance. I distinguish between 'indeterminacy' and 'chance', regardless of whether Peirce meant, in the last sentence, to distinguish or to identify these two concepts. There are indeed 'natural laws of chance' such as are familiar from quantum mechanics. For instance, Dirac [7, §2] explains that a polarized photon impinging on a polarizing crystal has a known probability of transmission, depending on the angle between the crystal axis and the plane of polarization. I don't know whether the next photon will be absorbed or transmitted, but I do know the chance of each outcome. In contrast, indeterminacy is a condition in which either I know no law, or no law is extant to govern the outcome. Indeterminacy is a more severe form of uncertainty than that represented by any probability distribution [2, chap. 7], [5].

Info-gap models of uncertainty quantify indeterminacy. Peirce's 'evolution' entails the initial existence of indeterminacy and its gradual disappearance by the spontaneous and unpredictable creation of law. Fallibilism establishes limitations on the discovery of laws. I will illustrate this by discussing indeterminism in economics for which specific examples of Peircean evolution of law can be adduced.

Theoretical economists are generally proud to assert that their discipline is epistemologically identical to physical science. Friedman is explicit [9, pp.4–5]. Samuelson is implicit by ascribing to economics the axiomatic structure of physics [26, pp.233–234]. Koopmans would also agree [15, pp.134–135].

In contrast, Habermas [12] emphasizes the non-nomological nature of social science, and Nelson and Winter stress that in evolutionary economics "things always are changing in ways that could not have been fully predicted" [17, p.370]. I will not review the rich literature, but only focus on what I will refer to as Shackle-Popper indeterminism. This idea was developed separately and in different ways by Shackle [27, pp.3–4, 156, 239, 401–402] and Popper [25, pp.80–81, 109].

The basic idea is that the behavior of intelligent learning systems displays an element of unstructured and unpredictable indeterminism. By 'intelligence' I mean: behavior is influenced by knowledge. This is surely characteristic of humans individually and of society at large. By 'learning' I mean a process of discovery: finding out today what was unknown yesterday. One economically important example of learning is what Keynes referred to as hearing 'the news'. Finally, indeterminism arises as follows: because tomorrow's discovery is by definition unknown today, tomorrow's behavior is not predictable today, at least not in its entirety. Given the richness of future discovery, (or its corollary, the richness of our current ignorance), the indeterminism of future behavior is broad, deep and unstructured. In Peirce's language, the laws of economic behavior will 'grow' or evolve in time as the agents in the economy make discoveries. These laws cannot be known ahead of time. Indeed, they don't exist at all until they emerge, since by definition discoveries cannot be predicted and the laws of economic behavior depend on the discoveries to be made.

Peircean indeterminate evolution has important practical consequences for mathematical modelling of social systems. Complexity and dimensionality are severe challenges in themselves. However, here we are dealing with the limited ability of laws, derived from *past* behavior, to describe *future* behavior. Peirce's notion of evolution explains why such laws are highly fallible. Intelligent learning behavior, as we have defined it, entails an element of spontaneous innovation resulting from discovery which seems to explain the painful experience of social modellers. Shackle-Popper indeterminism, which is a special case of Peircean fallibilism and evolution, accounts for the partially non-nomological nature of social systems. Peircean evolution is responsible for the inability of economic models to predict the structural changes which dominate the history of economic and social behavior. Peirce's anticipation of the asymptotic disappearance of indeterminism could arise, in the Shackle-Popper model, from the gradual exhaustion of the possibilities for new discoveries. From all appearances, we have a long way yet to go.

While Shackle-Popper indeterminism refers to intelligent learning systems it by no means presumes conscious self-awareness. For example, though the agents of an economy are consciously self-aware, the economy (which is what evolves) has no such attributes. Shackle-Popper indeterminism applies equally to biological evolution which proceeds by a process of self-invention: the characteristics of future eco-systems depend in no small measure on future phenotypic innovations. Gould makes this point very clearly in discussing the pre-Cambrian explosion. While we can retrospectively understand and reconstruct the logic of phylogenetic history, we could not have predicted which of the twenty-odd pre-Cambrian phyla would survive [10]. The unfolding of biological history determines both the possibilities and the impossibilities of that history itself.

And this brings us to Peirce's concept of mind. I leave the Peirce-experts to decipher Peirce's meaning. I will simply suggest that the evolutionary indeterminism described by Shackle and Popper is a special case of Peirce's contention that all things are 'mind' in one or another stage of evolution ([22] or [23, p.359]). If Peirce's model of mind can be sustained, then the social and natural sciences are grappling with different parts of the same evolving phenomenon and their methodologies are much the same.

## 4 Haack and the Info-gap Immunities

Haack's extensive debt to Peirce is admirably outlined by Haack herself in many places [1993.m, 1994.a3, 1996.a14, 2003.m, 11]. Since we have established that info-gap ideas can be found (at least implicitly) in Peirce's thought in many ways, it is not surprising that we find them in Haack's work as well. Peirce's principle of continuity, and the central role of non-probabilistic info-gap indeterminism which results, reverberate in Haack's foundherentist[4] theory of warrant.

I have read Haack's foundherentism with the following questions in mind. How could this theory of warrant be quantified and implemented in model-based decision theory? Conversely, and in particular, does foundherentism provide an epistemological framework for info-gap decision theory? A full treatment of these questions is beyond the scope of this article. All I intend is to demonstrate that the seeds of both of the info-gap decision functions can be found (lurking surreptitiously) in Haack's work. I must stress again that I do not claim that my interpretation is what Haack herself meant. I simply claim that the info-gap ideas to which I will refer are consistent with hers, and could be developed in a foundherentist context.

So here we go. In section 2 we discussed the info-gap robustness of the assertion that evidence $E^\star$ implies hypothesis $H$. 'Evidence' refers to data, models, understanding etc. The evidence in hand, $E^\star$, has info-gaps: the data are noisy and could have been different, the models could be wrong in ways we don't yet fathom, our understanding is incomplete, etc. We have an info-gap model for these uncertainties: an unbounded family of nested sets of possible bundles of evidence $E$. The sets are centered on and expand around the evidence in hand $E^\star$. The 'robust-satisficing' measure of warrant for the inference is: if $E^\star$ provides at least satisfactory support for $H$ and if any possible bundle of unactualized evidence $E$ up to a large horizon of uncertainty does so as well, then '$E^\star$ implies $H$' is highly warranted. In short, robust-satisficing warrant means: 'adequate support' and 'high robustness' imply high warrant. This is Peircean and synechistic because the basis for the warrant is that $E^\star$ is deeply embedded in a continuous and connected layer of potential but unrealized bundles of evidence.

I will now suggest a very different type of info-gap criterion of warrant, which I will call 'opportune-windfalling'. I caution the reader that opportune-windfalling does not necessarily merit the 'Good Housekeeping' seal of approval. It is even a bit eccentric. It nonetheless can sometimes be useful and it is always better to have than not. We will find it to be very Peircean and it is hinted at by Haack, perhaps subliminally.

We wish to test hypothesis $H$ and we have adequately supportive evidence $E^\star$ whose uncertainty is quantified by an info-gap model. Let $E$ be a different bundle of evidence which, if we had $E$, would provide overwhelming support for the hypothesis $H$. If we had $E$ this would be a 'wonderful windfall'. Now suppose that $E^\star$ is very similar to $E$, that only small errors in $E^\star$ could account for us having $E^\star$ rather than $E$. Since $E^\star$ is fraught with info-gaps, and since $E^\star$ is very close to $E$, we credit $H$ with at least some of the weight of $E$ by virtue of the proximity of $E^\star$ to $E$. $H$ is highly warranted in the sense of 'opportune-windfalling'. Concisely:

> The opportuneness of the assertion '$E^\star$ implies $H$' is measured as the lowest horizon of uncertainty (in the info-gap model) at which at least one bundle of evidence $E$ overwhelmingly supports $H$.

What the opportune-windfalling inquirer does, in seeking truly warranted assertions, is to seek hypotheses for which overwhelming support is very close to the evidence in hand. Again synechism is central to warrant: close connectivity between $E^\star$ and $E$ is the basis for opportune-windfalling warrant.

Robustness and opportunity are properties of assertions such as '$E^\star$ implies $H$'. The assertion is robust if $E^\star$ could err greatly without jeopardizing the adequate support for $H$. The assertion is opportune if minor modifications of $E^\star$ could result in overwhelming support for $H$. As such, robustness and opportunity are distinct measures of the warrant for $H$. Robust-satisficing and opportune-windfalling are both decision strategies. If $H$ is a statement of fact (e.g. about snarks) or of theory (e.g. about cosmology), then deciding to assent to $H$ can hinge on the strength of warrant for $H$. Alternatively, if $H$ entails an action as in 'If the central bank lowers the interest rate then unemployment will decrease', then assenting to $H$ is tantamount to deciding to take a particular action. The robust satisficer decides to assent to an assertion, which may imply an action, if the robust-satisficing warrant is strong. The opportune windfaller decides to assent if the opportune-windfalling warrant is strong.

Here we come to an interesting connection between Haack's foundherentism and info-gap robust-satisficing and opportune-windfalling. Both foundherentism and info-gap decision theory provide measures of warrant. Three questions must be addressed.

First, in endnote 1 I claimed that the 'adequate support' entailed in an info-gap robust test could be provided by foundherentism. So what is the structural relation between foundherentist warrant and info-gap robust-satisficing warrant? They are obviously different as evidenced by the fact that 'adequate support' in info-gap robust-satisficing warrant can be provided by many different epistemological theories: Haack's foundherentism, Thagard's idea of "coherence judgments that maximize constraint satisfaction" [28, pp.276–277], probabilistic decision theory [6, section 5], etc. In this sense, info-gap decision theory supervises any available operational warrant-based decision algorithm. On the other hand the hierarchical relation can be inverted. A foundherentist assessment of warrant, especially Haack's requirement for comprehensiveness, could be assessed by info-gap robust-satisficing based on some other (e.g. probabilistic) measure of adequate support. In this way robust-satisficing becomes one element in foundherentist warrant.

The treatment of uncertainty reveals a second connection between foundherentist and info-gap robust-satisficing concepts of warrant. Ideas of uncertainty are latent and implicit in Haack's theory of warrant. However, Haack distances herself strongly from probabilistic concepts [2003.m, pp.75–76]. I suggest that info-gap models provide one possible interpretation, via Peircean synechism and fallibilism, of Haack's conception of uncertainty in warrant from evidence. In info-gap theory these ideas of uncertainty are explicit, providing the central motivation for the theory as a whole.

The third question is: what is the structural relation between foundherentist warrant and info-gap opportune-windfalling warrant? In the first point we explained the flexible hierarchical relation of mutual support between foundherentism and robust-satisficing: either one can serve as a supporting element in implementing the other. Does this also hold for foundherentism and opportune-windfalling? Clearly, foundherentist warrant can provide the assessment of 'overwhelming support' entailed in an opportune-windfalling judgment, just as it can provide 'adequate support' for robust-satisficing. But can the reverse relation also be sustained? Can info-gap opportune-windfalling serve as an element in foundherentist warrant just as robust-satisficing can? Since opportune-windfalling is at least a little bit kinky, we must consider more closely whether any semblence of it can be found in foundherentism.

The very idea of opportune-windfalling may be repugnant to one steeped in robust-satisficing or other more conventional decision paradigms. What if $E^\star$ is very close to another bundle $E'$ which contradicts $H$ strongly? Granted; opportune windfalling is not necessarily robust. Tiny changes in the evidence can be very damaging: blood type A+ rather than A−, deleting a single nucleotide in the DNA sequence, etc. 'Tiny' doesn't mean 'insignificant'.[5] Opportune windfalling is not for the weak at heart. It is a gamble to buy into $H$ just because $E^\star$ is so close to $E$. But if one wants to (or must) bet on a theory, then opportune-windfalling can lend warrant and it can never detract.

The strangeness and questionableness of opportune-windfalling is the very point here: it is conceptually distinct from robust-satisficing. Whether one likes opportune-windfalling or not, it cannot hurt the status of $H$. In addition it cannot be derived from robust-satisficing. One of the important theorems of info-gap theory is that changes in the evidence which enhance the robust-satisficing support for $H$ may either enhance or detract from the opportune-windfalling support. The two measures of warrant are 'orthogonal'. (This is particularly obvious in the mathematical theory where the robustness and opportunity functions are anti-symmetric transforms of one another. The reader is encouraged to consult [4], section 3.1.) So are there echoes of windfalling in foundherentism?

Haack uses an argument in support of foundherentism which combines, in mutual support, elements which are reminiscent of both robust-satisficing and opportune-windfalling, though of course without reference to info-gap uncertainty. She summarizes one of several arguments thus:

> So the more justified A is in believing that p, *the closer his evidence is to all the relevant evidence,* and ... *the less room his evidence leaves for rivals to* p. (Italics in the original) [1993.m, p.217]

The first clause in italics is reminiscent of opportune-windfalling warrant, while the second clause suggests robust-satisficing. What is claimed in the first clause in italics can be interpreted approximately as follows. The evidence in hand is $E^\star$, while overwhelming evidence ("all the relevant evidence") is $E$. $E$ would be overwhelmingly supportive if we had it rather than just $E^\star$. The first clause asserts that close proximity of $E^\star$ to $E$ strongly justifies A's belief in proposition p. This is quite like opportune-windfalling warrant which is strong if proximity to overwhelming evidence is close, regardless of possible proximity to contradictory evidence. The metaphor in the second clause ("less room ... for rivals") is suggestive of robust-satisficing warrant: large robustness means that vast change in the evidence is needed to adequately support the rival hypothesis; there is "no room" for the alternative hypothesis. Robust satisficing warrant is strong if the robustness to error in the evidence is great.

This is not a explication of what Haack meant. She evidently intended the two italicized phrases to be different illustrations of the same idea. However it seems that both robust-satisficing and opportune-windfalling can play supporting roles in foundherentist warrant. Precisely how this is to be done in the case of windfalling is an open question. Likewise it is unclear what are the epistemological implications of two orthogonal info-gap measures of warrant — robustness and opportunity — coalescing in the single theory of foundherentism. Both types of info-gap warrant are deeply rooted in the structure of info-gap uncertainty. They both depend on the synechistic structure of uncertainty which is inherent in an info-gap model. So if info-gap robust-satisficing fits as a quantification of Peirce's conception of fallibilism, should we be surprised to find a hint of both robust-satisficing and opportune-windfalling in Haack's discussion of warrant?

I conclude with a final philosophical challenge. Practical decision theories tend to be eclectic, incorporating all conceptual tools which in one or another circumstance could prove useful. Furthermore, attempts to develop comprehensive theories of information and decision have been impressive, but not conclusive. For instance it is yet to be resolved whether "generalized information theory" is sufficiently general to encompass info-gap decision theory [13, p.37]. The case in point here has been the discussion of two distinct info-gap decision functions, the immunity functions for robustness and opportunity. These engender different and sometimes conflicting decision rules. At least some decision theorists who are involved in practical decision-making find no fault with an array of logically disparate conceptual tools. The philosophical question arising from this intellectual pluralism is: what decision procedures and concepts of warrant can *sometimes* be truth-indicative? This is logically distinct from the more traditional epistemological question: what procedures of inference are (possibly in some qualified sense) *always* truth-indicative? Decision analysts are often interested in those procedures falling between the extremes of universal validity and universal uselessness.

## Acknowledgements

The author is pleased to acknowledge useful comments and constructive criticisms by Professors Mark Burgman (Melbourne), Susan Haack (Miami), Henry Kyburg (Rochester), Iddo Landau (Haifa), Jan Michl (Oslo) and Paul Thagard (Waterloo).

# 5 References

1. Ben-Haim, Yakov, 1994, Convex models of uncertainty: applications and implications, *Erkenntnis,* 41: 139–156.

2. Ben-Haim, Yakov, 1996, *Robust Reliability in the Mechanical Sciences,* Springer-Verlag, Berlin.

3. Ben-Haim, Yakov, 1999, Set-models of information-gap uncertainty: Axioms and an inference scheme, *Journal of the Franklin Institute,* 336: 1093–1117.

4. Ben-Haim, Yakov, 2006, *Info-Gap Decision Theory: Decisions Under Severe Uncertainty,* 2nd edition, Academic Press, London.

5. Ben-Haim, Yakov, 2004, Uncertainty, probability and information-gaps, *Reliability Engineering and System Safety,* 85: 249–266.

6. Ben-Haim, Yakov, 2004, Info-gap Decision Theory For Engineering Design. Or: Why 'Good' is Preferable to 'Best', to appear as chapter 11 in *Engineering Design Reliability Handbook,* Edited by E. Nikolaides and D. Ghiocel, CRC Press.

7. Dirac, P.A.M., 1958, *The Principles of Quantum Mechanics,* 4th ed., Oxford University Press.

8. Dubois, Didier and Henri Prade, 1986, *Possibility Theory: An Approach to Computerized Processing of Uncertainty.* With the collaboration of H. Farreny, R. Martin-Clouaire and C. Testemale. Translated by E.F. Harding, Plenum Press, New York.

9. Friedman, Milton, 1953, On the methodology of positive economics, In: Friedman, Milton, *Essays in Positive Economics,* University of Chicago Press.

10. Gould, Stephen Jay, 1989, *Wonderful Life: The Burgess Shale and the Nature of History,* W.W. Norton and Co., New York.

11. Haack, Susan, 2004, Not cynicism but synechism: Lessons from classical pragmatism, to appear in *Companion to Pragmatism,* ed. by Joseph Margolis and John Shook, Blackwell.

12. Habermas, Jürgen, 1970, *On the Logic of the Social Sciences,* trans. by Shierry Weber Nicholsen and Jerry A. Stark, 1990, MIT Press.

13. Klir, George J., 2004, Generalized information theory: aims, results and open problems, *Reliability Engineering and System Safety,* 85: 21–38.

14. Kolmogorov, A.N., 1933, *Foundations of the Theory of Probability,* 2nd English edition translated by N. Morrison, based on the 1933 German monograph.

15. Koopmans, Tjalling C., 1957, *Three Essays on the State of Economic Science,* McGraw-Hill Book Co., New York.

16. Moore, Walter, 1989, *Schrodinger: Life and Thought*, Cambridge University Press.

17. Nelson, Richard R. and Sidney G. Winter, 1982, *An Evolutionary Theory of Economic Change,* Belknap Press, Harvard University.

18. Peirce, Charles Sanders, 1878, The Doctrine of Chances, *Popular Science Monthly.* Reprinted in [24], pp.61–81.

19. Peirce, Charles Sanders, 1891, The architecture of theories, *The Monist.* Reprinted in [24], pp.157–178.

20. Peirce, Charles Sanders, 1897, *Collected Papers,* 1.170, 171-175. Reprinted in [23] pp.355-358.

21. Peirce, Charles Sanders, 1902, 'Synechism', in Baldwin's *Dictionary of Philosophy and Psychology.* Reprinted in [23], pp.354–355.

22. Peirce, Charles Sanders, 1902, 'Uniformity', in Baldwin's *Dictionary of Philosophy and Psychology.* Selection reprinted in [23], pp.359–360.

23. Peirce, Charles Sanders, 1955, *Philosophical Writings of Peirce.* Justus Buchler, editor. Dover reprint of *The Philosophy of Peirce: Selected Writings,* Routledge and Kegan Paul, 1940.

24. Peirce, Charles Sanders, 1998, *Chance, Love, and Logic: Philosophical Essays,* ed. by M.R. Cohen, University of Nebraska Press.

25. Popper, Karl, 1982, *The Open Universe: An Argument for Indeterminism.* From the Postscript to *The Logic of Scientific Discovery,* Routledge.

26. Samuelson, P.A., 1963, Problems of methodology — discussion. *American Economic Review,* Papers and Proceedings of the 75th Meeting of the American Economic Association, May 1963, 53: 231–236.

27. Shackle, G.L.S., 1972, *Epistemics and Economics: A Critique of Economic Doctrines,* Transaction Publishers, 1992, originally published by Cambridge University Press.

28. Thagard, Paul, 2000, *Coherence in Thought and Action,* MIT Press.

29. Thompson, D'Arcy Wentworth, 1966, *On Growth and Form,* Abridged and edited by J.T. Bonner, Cambridge University Press.

**Endnotes**

[1] I mean "adequate" in distinction from "overwhelmingly strong" support. What constitutes "adequate support" is a difficult epistemological question. From the point of view of an info-gap inference we are free to supply alternative criteria, understanding that different criteria may have different levels of credence. One very comprehensive and convincing criterion for adequacy of evidence is Haack's "foundherentist" theory of warrant [1993.m, 2001.a1]. We will return to Haack later.

[2] Erwin Schrodinger once complained to Max Born: "All is linear, linear — linear in the $n$th power I would say, if that was not a contradiction." Quoted in [16], p.381.

[3] Testing a linear model is different from testing the assertion that non-linear terms, say quadratic or cubic terms, are small or zero.

[4] Haack's fondness for unpronounceable neologisms is an early indication that she carries a Peircean gene.

[5] Robert Browning: "Oh! the little more, and how much it is!/ And the little less, and what worlds away."