Lecture 1

Info-Gap Theory:

Overview and Examples

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1 *Highlights*

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§ **What is an info-gap?** (Uncertainty is unbounded)

§

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§ **Examples**

2 *Info-Gap Uncertainty: Examples*

∼∼*Thames Flood Barrier*∼∼

Figure 1: **1953 barrier breach.**

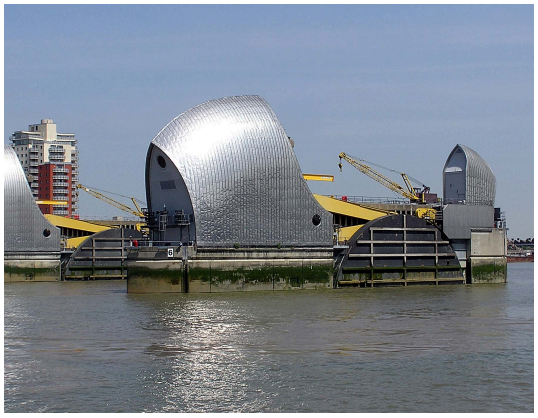

Figure 2: **Barrier element.**

§ Some facts:

- **1953: worst storm surge of century.**
- **Flood defences breached.**
- **307 dead. Thousands evacuated.**
- **Canvey Island in Estuary devastated.**
- **Current barrier opened May 1984.**

§ Thames 2100:

Major re-design of flood defences.

§ Uncertainties:

- Statistics of surge height:
  - Fairly complete: most years since 1819.
  - Planning for 1000-year surge.
- Global warming: sea level rise.
- Tectonic settling of s. England.
- Damage vs flood depth.
- Human action: dredging, embanking.
- Urban development.

§ Severe Knightian uncertainties: Gaps in knowledge, understanding and goals.

∼∼*Fukushima Nuclear Reactor*∼∼

Figure 3: **Sea wall breach.**

Figure 4: **Hydrogen explosion.**

§ Some facts:

- **11.3.2011: Richter-9 earthquake in NE Japan.**
- **Tsunami followed shortly.**
- **Sea wall breached: fig. 3.‡**
- **Hydrogen explosion several days later. Fig. 4.‡**
- **Slow disaster recovery.**

§ Info-gaps:

- **Sub-system interactions.**
- **Institutional constraints.**

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‡ http://www.dailymail.co.uk/news/article-1388629/Japan-tsunami-destroyed-wall-designed-protect-Fukushima-nuclear-plant.html

∼∼*Managing Mobile Wireless Network*∼∼

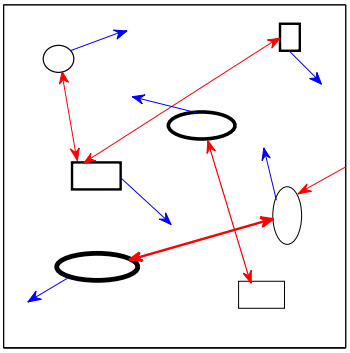

Figure 5: **Mobile wireless network.**
Red: talk. Blue: motion.

- **Manage resources.**
- **Info-gaps:**
  - **Node number, motion, transmission.**
  - **Barriers.**
  - **Cross talk.**

∼∼*Climate Change*∼∼

§ **The issue:**

Sustained rise in green house gases

results in temperature rise

which results in adverse economic impact.

§ **Models:**

- **Temperature change:** $\Delta \mathbf{CO}_2 \Longrightarrow \Delta T$.
- **Economic impact:** $\Delta T \Longrightarrow \Delta \mathbf{GDP}$.

§ **The problems:**

- **Models highly uncertain.**
- **Data controversial.**

§ E.g., IPCC model for

Uncertainty in Equil'm Clim. Sensi'ty, S.

- **Likely range: 1.5°C to 4.5°C.**
- **Extreme values highly uncertain.**
- **95th quantile of S in 10 studies:**

  Mean: 7.1°C. St. Dev: 2.8°C.

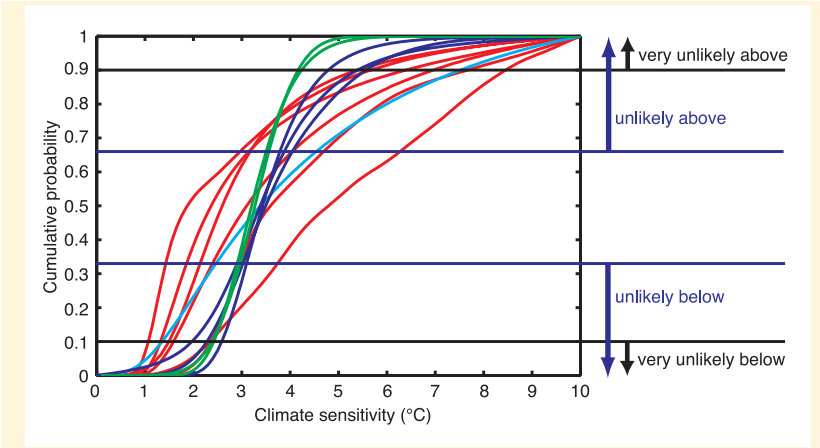

Figure 6: IPCC ch.10, p.799.

∼∼*Summary*∼∼

§ **Severe Knightian uncertainties: Gaps in knowledge, understanding and goals.**

§

∼∼*Summary*∼∼

§ Severe Knightian uncertainties: Gaps in knowledge, understanding and goals.

§ Info-Gap models of uncertainty:

- Disparity between what is known and what needs to be known for responsible decision.
-

∼∼*Summary*∼∼

§ Severe Knightian uncertainties: Gaps in knowledge, understanding and goals.

§ Info-Gap models of uncertainty:

- Disparity between what is known and what needs to be known for responsible decision.
- Unbounded family of sets of events (points, functions or sets).
- No known worst case.
- No funcs. of probability, plausibility, likelihood, etc.
- Hybrid: info-gap model of probabilities.

3 *Principle of Indifference*

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§ The info-gap contention:

The probabilistic domain of discourse
does not encompass all epistemic uncertainty.

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§ The info-gap contention:

The probabilistic domain of discourse
does not encompass all epistemic uncertainty.

§ We will consider common misuses of probability.

3.1 *Keynes' Example*

§ ρ = **specific gravity** [g/cm^3] **is unknown:**

$$1 \leq \rho \leq 3$$

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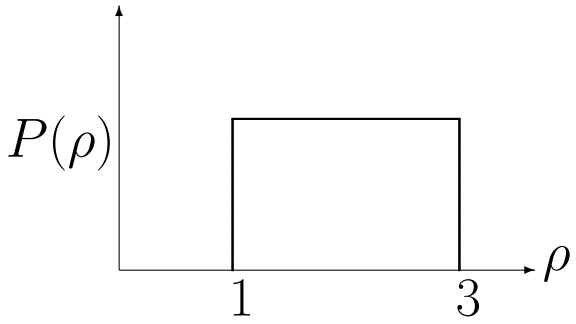
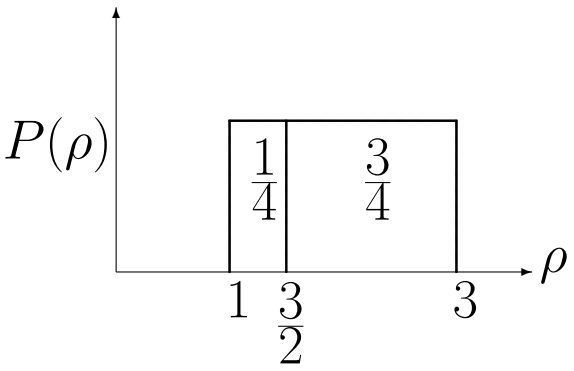
§ **Principle of indifference:**

Uniform distribution in $[1,\ 3]$**, so:**



§ Uniform distribution in [1, 3], so:

$$\textbf{Prob}\left(\frac{3}{2} \leq \rho \leq 3\right) = \frac{3}{4}$$



§ ϕ = **specific volume** $[\mathrm{cm}^3/\mathrm{g}]$ **is unknown:**

$$\frac{1}{3} \leq \phi \leq 1$$

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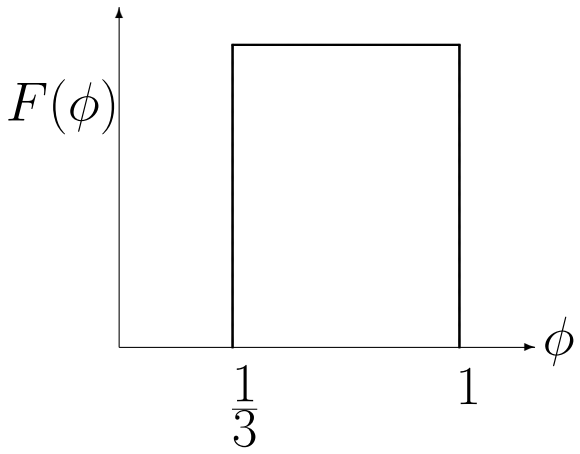
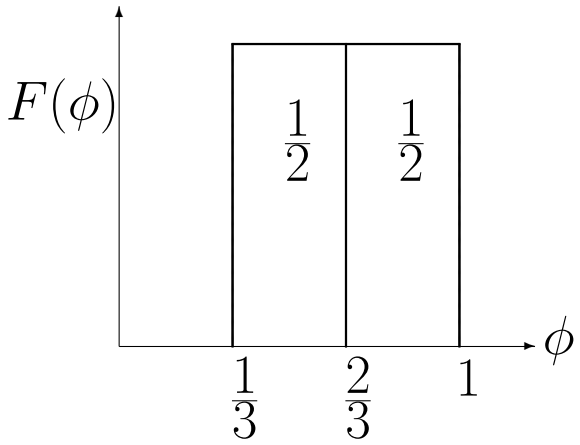


§ Principle of indifference:

Uniform distribution in $\left[\frac{1}{3},\ 1\right]$, so:

$$\textbf{Prob}\left(\frac{1}{3} \leq \phi \leq \frac{2}{3}\right) = \frac{1}{2}$$



§ **These two events are identical:**

$$\underbrace{\left(\frac{1}{3} \leq \phi \leq \frac{2}{3}\right)}_{\textbf{Specific volume}} \equiv \underbrace{\left(\frac{3}{2} \leq \rho \leq 3\right)}_{\textbf{Specific gravity}} \tag{1}$$

§

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§ **Hence their probabilities are equal:**

$$\underbrace{\textbf{Prob}\left(\frac{1}{3} \leq \phi \leq \frac{2}{3}\right)}_{\textbf{Specific volume}} = \underbrace{\textbf{Prob}\left(\frac{3}{2} \leq \rho \leq 3\right)}_{\textbf{Specific gravity}} \tag{3}$$

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§ **Hence:** $\frac{1}{2} = \frac{3}{4}$

$$\frac{1}{2} = \underbrace{\mathbf{Prob}\left(\frac{1}{3} \leq \phi \leq \frac{2}{3}\right)}_{\text{Specific volume}} = \underbrace{\mathbf{Prob}\left(\frac{3}{2} \leq \rho \leq 3\right)}_{\text{Specific gravity}} = \frac{3}{4}$$

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§ **The Culprit: Principle of indifference.**

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§ **The Culprit: Principle of indifference.**

§ **Ignorance is not probabilistic. It's an info-gap.**

3.2 *2-Envelope Riddle*

§ The riddle:

- **You are presented with two envelopes.**
  - **Each contains a positive sum of money.**
  - **One contains twice the contents of the other.**
- **You choose an envelope, open it, and find $\$\,50$.**
- **Would you like to switch envelopes?**

§ You reason as follows:

- Other envelope contains either \$ 25 or \$ 100.
- Principle of indifference:
- Assume equal probabilities.

  The expected value upon switching is:

  $\mathbf{E.V.} = \frac{1}{2}\ \$\,25 + \frac{1}{2}\ \$\,100 = \$\,62.50.$

  $\$\,62.50 > \$\,50.$
- Yes! Let's switch, you say.

§ The riddle, re-visited:

- **You are presented with two envelopes.**
  - **Each contains a positive sum of money.**
  - **One contains twice the contents of the other.**
- **You choose an envelope, but do not open it.**
- **Would you like to switch envelopes?**

§ You reason as follows:

- This envelope contains \$ X > \$ 0.
- Other envelope contains either \$ $2X$ or \$ $\frac{1}{2}X$.
- Principle of indifference:
- Assume equal probabilities.

  The expected value upon switching is:

  E.V. $= \frac{1}{2}$ \$ $2X + \frac{1}{2}$ \$ $\frac{1}{2}X =$ \$ $\left(1+\frac{1}{4}\right)X > X$.
- Yes! Let's switch, you say.

§ **You reason as follows:**

- **This envelope contains** $\$X >\ \$0$.
- **Other envelope contains either** $\$2X$ **or** $\$\frac{1}{2}X$.
- **Principle of indifference:**
- **Assume equal probabilities.**

  The expected value upon switching is:

  $$\textbf{E.V.} = \tfrac{1}{2}\ \$2X + \tfrac{1}{2}\ \$\tfrac{1}{2}X = \$\left(1+\tfrac{1}{4}\right)X\ >\ X.$$
- **Yes! Let's switch, you say.**

§ **You wanna switch again? And again? And again?**

4 *Conclusion*

In Conclusion

§ **Info-gap uncertainty:**

innovation, discovery, ignorance, surprise.

§

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§ **Info-gap uncertainty is unbounded.**

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§ **Realism: our models are wrong now**

(and we don't know where or how much).

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In Conclusion

§ **Info-gap uncertainty:**

innovation, discovery, ignorance, surprise.

§ **Info-gap uncertainty is unbounded.**

§ **Optimism: our models get better all the time.**

§ **Realism: our models are wrong now**

(and we don't know where or how much).

§ **Responsible decision making:**

- **Specify your goals.**
- **Maximize your robustness to uncertainty.**
- **Study the trade offs.**
- **Exploit windfall opportunities.**