Info-Gap Theory
Concepts and Applications

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Outline

Risk or Uncertainty?
Probability is powerful, but ignorance is not probabilistic

Uncertainty and the optimization imperative
- Limits of prediction and outcome-optimization
- Robust satisficing

Time to Recovery: Innovation dilemma

Optimal monitoring and surveillance: A paradox
Risk and Uncertainty

Probabilistic risk

or

Knightian “true uncertainty”
## Probabilistic Risk

<table>
<thead>
<tr>
<th>Consequence</th>
<th>Probability</th>
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<tbody>
<tr>
<td>Drought</td>
<td>Stochastic process</td>
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<tr>
<td>Industrial accident</td>
<td>Actuarial tables</td>
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<tr>
<td>Tsunami</td>
<td>Historical data</td>
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<td>Faulty air filters</td>
<td>Quality control data</td>
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<tr>
<td>Deception, scam</td>
<td>Sociological data</td>
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</tbody>
</table>

**Risk is:**
- Structured: known event space
- Modeled with probability
- Manageable *(but still risky)*
Frank Knight’s “true uncertainty”

“The uncertainties which persist ... are **uninsurable** because there is no objective measure of the probability”.

- Fukushima
- Arab Spring
- Russia-Crimea
- ESP
- Diagnostic lens
- 3-d printed car
“We live on an island of knowledge surrounded by a sea of ignorance. As our island of knowledge grows, so does the shore of our ignorance.”

*John A. Wheeler*
Non-probabilistic true uncertainty

Discovery

- America
- Nuclear fission
- Martians (not yet?)
Non-probabilistic true uncertainty

D  Discovery

I  Invention/Innovation

- Printing press: material invention.
- Ecological responsibility: conceptual innovation.
- French revolution: social innovation.
Non-probabilistic true uncertainty

D  Discovery
I  Invention/Innovation
S  Surprise (Asymmetric uncertainty)

- Ambush
- Competitor’s innovation
- Natural catastrophe
Non-probabilistic true uncertainty

**D** Discovery

**I** Invention/Innovation

**S** Surprise (Asymmetric uncertainty)

What’s the next **D** **I** or **S** ???

Knightian uncertainty:

- Unstructured: unknown event space.
- Indeterminate: no laws.
- Barely manageable.
Glenn L. Shackle, 1903-1992
Karl Popper, 1902-1994
Shackle-Popper Indeterminism

**Intelligence:**
What people **know**, influences how they **behave**.

**Discovery:**
What will be **discovered tomorrow** can’t be **known today**.

**Indeterminism:**
Tomorrow's behavior can’t be fully modelled today.

- Info-gaps, indeterminism: **unpredictable**.
- Ignorance is not probabilistic.
Uncertainty and the Optimization Imperative

Doing your best:
What does that mean?
• Outcome optimization.
• Procedural optimization.

Implications for decision making:
Robust satisficing.
Substantive outcome optimization:
• Predict outcomes of available options.
• Select predicted best option.
Substantive outcome optimization.

Useful under risk:

• Structured uncertainty.
• Reliable probabilistic predictions.
Doing Your Best

Substantive outcome optimization:

Useful under risk.

Not useful (irresponsible?) under uncertainty.

- Unstructured uncertainty.
- Unreliable predictions.
Questions

What do we (not) know?

Robustness questions:
• What is an essential outcome?
• How to be robust to surprise?

Opportuneness questions:
• What is a windfall outcome?
• How to exploit opportunities?

How to prioritize decision options?

What are the trade offs?
Robustness answer:
- System model
- Outcome requirement
- Uncertainty model
- Robustness function
- Prioritized options

Opportuneness answer:
- System model
- Outcome aspiration
- Uncertainty model
- Opportuneness function
- Prioritized options
Robust Satisficing

Two questions for decision makers:
1. What are our goals?
2. How much error/surprise can we tolerate?
Robust Satisficing

Two questions for decision makers:
1. What are our goals?
2. How much error/surprise can we tolerate?

   • Essential goals.
   • Worst acceptable outcomes.
   • Modest or ambitious.
Robust Satisficing

Two questions for decision makers:

1. What are our goals?
2. How much error/surprise can we tolerate?


2. Robustness:
   - Immunity to ignorance.
   - Greatest tolerable error or surprise.
# Robust Satisficing

Two questions for decision makers:

1. What are our goals?
2. How much error/surprise can we tolerate?

1. **Satisficing**: Achieving critical outcomes.

2. **Robustness**: Greatest tolerable error.

Optimize robustness; satisfice goals:

   **Procedural** (not substantive) optimization.
Time To Recovery

Time to recovery (TTR) after disruption:

• Building after earthquake.
• Energy distribution network after failure.
• Micro-sensor after shock load.
• Etc.

Task: Recover critical functions in specified time.

Challenge: **Uncertainties** (info-gaps).
Time To Recovery

Formulation: Innovation dilemma.

• Choose between 2 design concepts:
  o State of the art (SotA, \( q=0 \)).
  o New and innovative (NaI, \( q=1 \)).

• System model: TTR, \( t(a,q) \), to load \( a \) for system \( q \).

• Outcome requirement and aspiration:
  \[
  t(a,q) < t_c, \quad t(a,q) < t_w (<< t_c)
  \]

Info-gaps:

• Parameter uncertainty: value of \( a \).
• Functional uncertainty: shape of \( t(a,q) \).
Time To Recovery

Estimated TTR functions for 2 designs.

Putative preference: NaI predicted better than SotA.

What about **uncertainty** in load $a$ & TTR func $t(a,q)$?
Time To Recovery

Info-gap:
Disparity between what we do know (on $a$ & $t(a,q)$) and what we need to know in order to make responsible decision (SotA or NaI).

About the load, $a$:
Known estimated value. Unknown fractional error.

About the TTR function, $t(a,q)$:
• Known estimated form. Unknown fractional error.
• NaI more uncertain than SotA.
Time To Recovery

Info-gap model of uncertain $a$ and $t(q,a)$:

$$\mathcal{U}(h) = \left\{ a, t_q(a) : t_q(a) \geq 0, \; |t_q(a) - \bar{t}_q(a)| \leq hw_q \bar{t}_q(a), \; q = 0, 1. \right\}$$

$$a > 0, \; \left| \frac{a - \bar{a}}{s} \right| \leq h \right\}, \; \; h \geq 0$$

- Non-prob: unbounded family of nested sets.
- Horizon of uncertainty, $h$, unknown.
- No known worst case.
- Axioms: Contraction and Nesting.
Time To Recovery

**Immunity functions.**

**Robustness: immunity against failure.**

Maximum tolerable uncertainty.

\[
\widehat{h}_q(t_c) = \max \left\{ h : \left( \max_{t,a \in \mathcal{U}(h)} t_q \right) \leq t_c \right\}
\]  \hspace{1cm} (21)

**Opportuneness: immunity against windfall.**

Minimum necessary uncertainty.

\[
\widehat{\beta}_q(t_w) = \min \left\{ h : \left( \min_{t,a \in \mathcal{U}(h)} t_q \right) \leq t_w \right\}
\]  \hspace{1cm} (22)
Time To Recovery

Robustness vs. Requirement

Trade off: better TTR means worse robustness.
Zeroing: Predicted TTR has zero robustness.
Preference reversal:
- NaI preferred at low TTR. SotA preferred at hi TTR.
- Resolution of innovation dilemma.
Zeroing: Predicted TTR possible without uncertainty.

Trade off: wonderful TTR needs more uncertainty.

No preference reversal:
- No crossing opportuneness curves.
- NaI more uncertain and more opportune.
<table>
<thead>
<tr>
<th>Time To Recovery: Summary</th>
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<tr>
<td><strong>Task:</strong> Recover critical functions in specified time.</td>
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<tr>
<td><strong>Info-gaps:</strong></td>
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<td>- Parameter uncertainty: value of $a$.</td>
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<td><strong>Innovation dilemma:</strong> NaI vs. SotA.</td>
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<td><strong>Robustness:</strong> maximum tolerable uncertainty.</td>
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<td><strong>Opportuneness:</strong> minimum required uncertainty.</td>
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<td><strong>Trade off, zeroing:</strong> robustness and opportuneness.</td>
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Optimal monitoring and surveillance: A paradox of learning

Learning:

- Discover new knowledge.
- **Not:** learn French or Newtonian Physics.

Optimal learning:

Min time, max quantity, min cost, max quality...

Monitoring and surveillance as learning:

- New failure mechanism emerging? Where? What?...
- **Not:** does this firm use that amount of power?
Optimal Learning: A Paradox

- Discover & prevent new failure with max effectivity.

- Range of design alternatives with fixed resources:
  - Extensive research: more knowledge, but less impact.
  - Limited research: less knowledge, but more impact.

- Optimal research amount depends on failure mechanism.
- Failure mechanism is unknown.

Resolution: Satisfice effectivity. Maximize robustness.

Procedural (not substantive) optimization.

Alternatives: Optimal adaptive or stochastic learning?

Same paradox of optimal learning.

Same resolution: robustly satisfice the design of the learning.
Summing Up

Risk or Uncertainty:

- **Probabilistic** risk, **Knightian** uncertainty (info-gaps).
- Shackle-Popper indeterminism.

Substantive outcome optimization:

Useful under **risk**, not under **uncertainty**.

Robust satisficing: Optimize robustness; satisfice goals.

- **Procedural** (not substantive) optimization.

Opportune windfalling: use propitious uncertainty.

Time to recovery: Innovation dilemma.

Optimal monitoring and surveillance: A paradox
Questions?