Lecture 1

Info-Gap Theory: Overview and Examples

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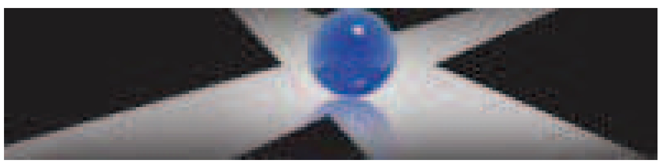

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1 *Highlights*

Highlights

§ **What is an info-gap? (Uncertainty is unbounded)**

§

Highlights

§ **What is an info-gap? (Uncertainty is unbounded)**

§ **Examples**

2 *Info-Gap Uncertainty: Examples*

∼∼*Thames Flood Barrier*∼∼

Figure 1: **1953 barrier breach.**

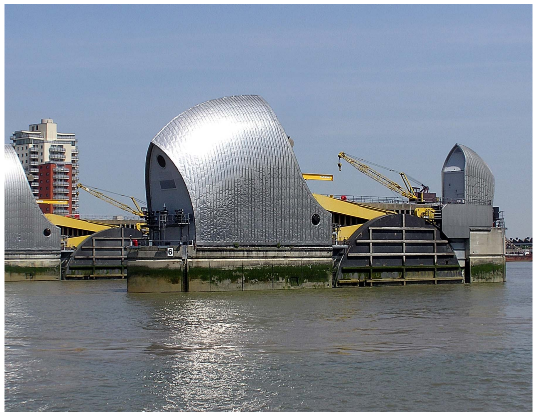

Figure 2: **Barrier element.**

§ Some facts:

- **1953: worst storm surge of century.**
- **Flood defences breached.**
- **307 dead. Thousands evacuated.**
- **Canvey Island in Estuary devastated.**
- **Current barrier opened May 1984.**

§ **Thames 2100:**

Major re-design of flood defences.

§ **Uncertainties:**

- **Statistics of surge height:**
  - **Fairly complete: most years since 1819.**
  - **Planning for 1000-year surge.**
- **Global warming: sea level rise.**
- **Tectonic settling of s. England.**
- **Damage vs flood depth.**
- **Human action: dredging, embanking.**
- **Urban development.**

§ **Severe Knightian uncertainties: Gaps in knowledge, understanding and goals.**

∼∼*Fukushima Nuclear Reactor*∼∼

Figure 3: **Sea wall breach.**

Figure 4: **Hydrogen explosion.**

§ Some facts:

- **11.3.2011: Richter-9 earthquake in NE Japan.**
- **Tsunami followed shortly.**
- **Sea wall breached: fig. 3.**‡
- **Hydrogen explosion several days later. Fig. 4.**‡
- **Slow disaster recovery.**

§ Info-gaps:

- **Sub-system interactions.**
- **Institutional constraints.**

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‡ http://www.dailymail.co.uk/news/article-1388629/Japan-tsunami-destroyed-wall-designed-protect-Fukushima-nuclear-plant.html

∼∼Managing Mobile Wireless Network∼∼

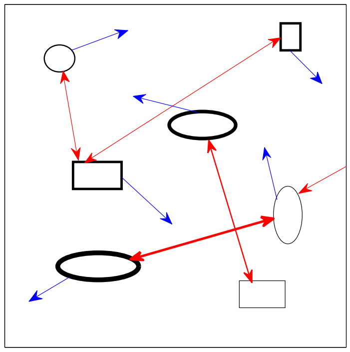

Figure 5: **Mobile wireless network.**
Red: talk. Blue: motion.

- **Manage resources.**

- **Info-gaps:**
  - **Node number, motion, transmission.**
  - **Barriers.**
  - **Cross talk.**

∼∼*Climate Change*∼∼

§ The issue:

Sustained rise in green house gases

results in temperature rise

which results in adverse economic impact.

§ Models:

- **Temperature change:** $\Delta \mathrm{CO}_2 \Longrightarrow \Delta T$.
- **Economic impact:** $\Delta T \Longrightarrow \Delta \mathrm{GDP}$.

§ The problems:

- **Models highly uncertain.**
- **Data controversial.**

§ E.g., IPCC model for

Uncertainty in Equil'm Clim. Sensi'ty, S.

- **Likely range: 1.5$^{\mathrm{o}}$C to 4.5$^{\mathrm{o}}$C.**
- **Extreme values highly uncertain.**
- **95th quantile of S in 10 studies:**

  Mean: 7.1$^{\mathrm{o}}$C. St. Dev: 2.8$^{\mathrm{o}}$C.

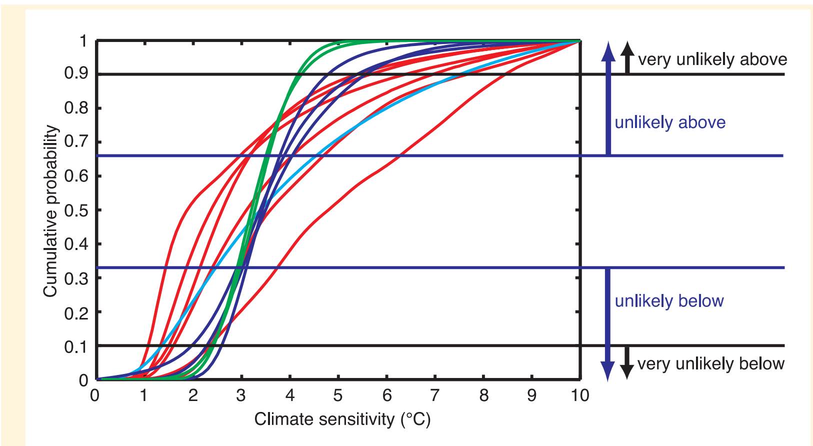

Figure 6: IPCC ch.10, p.799.

~~*Summary*~~

§ **Deep Knightian uncertainties: Gaps in knowledge, understanding and goals.**

§

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§ Deep Knightian uncertainties: Gaps in knowledge, understanding and goals.

§ Info-Gap models of uncertainty:

- **Disparity between what is known and what needs to be known for responsible decision.**
-

∼∼*Summary*∼∼

§ **Deep Knightian uncertainties: Gaps in knowledge, understanding and goals.**

§ **Info-Gap models of uncertainty:**

- **Disparity between what is known and what needs to be known for responsible decision.**
- **Unbounded family of sets of events (points, functions or sets).**
- **No known worst case.**
- **No functions of probability, plausibility, likelihood, etc.**
- **Hybrid: info-gap model of probabilities.**

3 *Principle of Indifference*

§ Question: Is ignorance probabilistic?

§

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The probabilistic domain of discourse
does not encompass all epistemic uncertainty.

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  are assigned equal probabilities.
- **Uniform distribution represents complete ignorance.**

§ **The info-gap contention:**

The probabilistic domain of discourse
does not encompass all epistemic uncertainty.

§ **We will consider common misuses of probability.**

3.1 *Keynes' Example*

§ **ρ = specific gravity [g/cm^3] is unknown:**

$$1 \;\leq\; \rho \;\leq\; 3$$

§

§ ρ = **specific gravity [g/cm^3] is unknown:**

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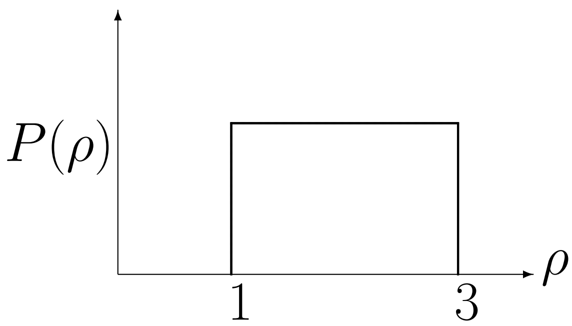
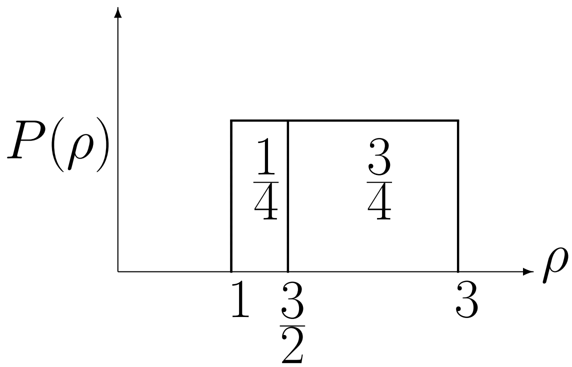
§ **Principle of indifference:**

Uniform distribution in $[1, \; 3]$, so:



§ Uniform distribution in $[1, \ 3]$, so:

$$\mathbf{Prob}\left(\frac{3}{2} \ \le \ \rho \ \le \ 3\right) = \frac{3}{4}$$



§ ϕ = **specific volume [cm^3/g] is unknown:**

$$\frac{1}{3} \le \phi \le 1$$

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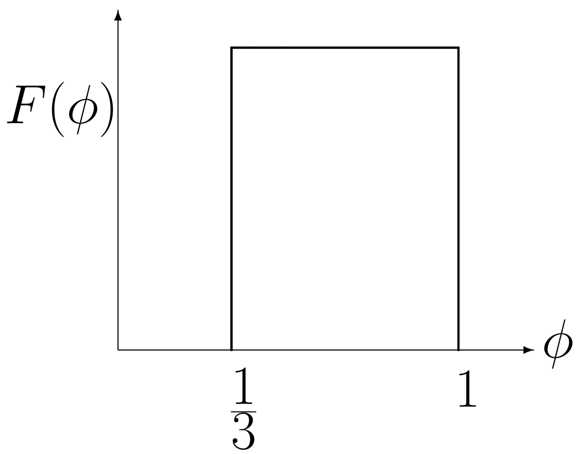
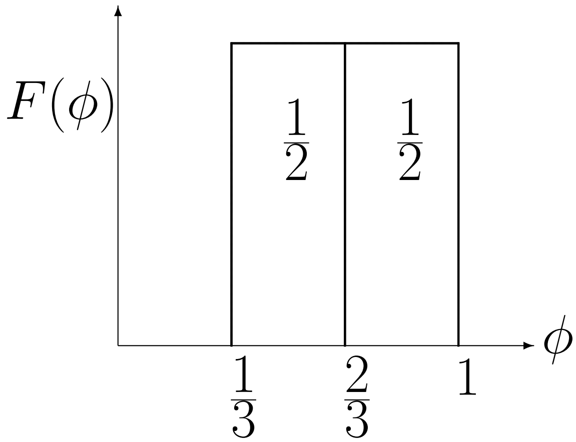


§ Principle of indifference:

Uniform distribution in $\left[\frac{1}{3},\ 1\right]$, so:

$$\textbf{Prob}\left(\frac{1}{3} \le \phi \le \frac{2}{3}\right) = \frac{1}{2}$$



§ These two events are identical:

$$\underbrace{\left(\frac{1}{3} \;\leq\; \phi \;\leq\; \frac{2}{3}\right)}_{\textbf{Specific volume}} \equiv \underbrace{\left(\frac{3}{2} \;\leq\; \rho \;\leq\; 3\right)}_{\textbf{Specific gravity}} \tag{1}$$

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§ Hence their probabilities are equal:

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§ Hence: $\frac{1}{2} = \frac{3}{4}$

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§ **These two events are identical:**

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§ **Hence their probabilities are equal:**

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§ **The Culprit: Principle of indifference.**

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$$\frac{1}{2} = \underbrace{\textbf{Prob}\left(\frac{1}{3} \;\leq\; \phi \;\leq\; \frac{2}{3}\right)}_{\textbf{Specific volume}} = \underbrace{\textbf{Prob}\left(\frac{3}{2} \;\leq\; \rho \;\leq\; 3\right)}_{\textbf{Specific gravity}} = \frac{3}{4}$$

§ **The Culprit: Principle of indifference.**

§ **Ignorance is not probabilistic. It's an info-gap.**

3.2 *2-Envelope Riddle*

§ The riddle:

- **You are presented with two envelopes.**
  - **Each contains a positive sum of money.**
  - **One contains twice the contents of the other.**
- **You choose an envelope, open it, and find \$ 50.**
- **Would you like to switch envelopes?**

§ **You reason as follows:**

- **Other envelope contains either $\$\,25$ or $\$\,100$.**
- **Principle of indifference:**
- **Assume equal probabilities.**

  The expected value upon switching is:

  E.V. $= \frac{1}{2}\ \$\,25 + \ \frac{1}{2}\ \$\,100 = \$\,62.50$.

  $\$\,62.50 > \ \$\,50$.

- **Yes! Let's switch, you say.**

§ The riddle, re-visited:

- **You are presented with two envelopes.**
  - **Each contains a positive sum of money.**
  - **One contains twice the contents of the other.**
- **You choose an envelope, but do not open it.**
- **Would you like to switch envelopes?**

§ You reason as follows:

- **This envelope contains $\$\,X > \$\,0$.**
- **Other envelope contains either $\$\,2X$ or $\$\,\frac{1}{2}X$.**
- **Principle of indifference:**
- **Assume equal probabilities.**

  The expected value upon switching is:

  E.V. $= \frac{1}{2}\,\$\,2X + \frac{1}{2}\,\$\,\frac{1}{2}X = \$\left(1+\frac{1}{4}\right)X > X.$
- **Yes! Let's switch, you say.**

§ **You reason as follows:**

- **This envelope contains \$ $X >$ \$ 0.**
- **Other envelope contains either \$ $2X$ or \$ $\frac{1}{2}X$.**
- **Principle of indifference:**
- **Assume equal probabilities.**

  The expected value upon switching is:

  E.V. $= \frac{1}{2}$ **\$** $2X + \frac{1}{2}$ **\$** $\frac{1}{2}X =$ **\$** $\left(1 + \frac{1}{4}\right)X > X$.
- **Yes! Let's switch, you say.**

§ **You wanna switch again? And again? And again?**

4 *Conclusion*

In Conclusion

§ **Info-gap uncertainty:**

innovation, discovery, ignorance, surprise.

§

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§ **Info-gap uncertainty is unbounded.**

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§ **Realism: our models are wrong now (and we don't know where or how much).**

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In Conclusion

§ **Info-gap uncertainty:**
innovation, discovery, ignorance, surprise.

§ **Info-gap uncertainty is unbounded.**

§ **Optimism:** **our models get better all the time.**

§ **Realism:** **our models are wrong now**
(and we don't know where or how much).

§ **Responsible decision making:**

- **Specify your goals.**
- **Maximize your robustness to uncertainty.**
- **Study the trade offs.**
- **Exploit windfall opportunities.**