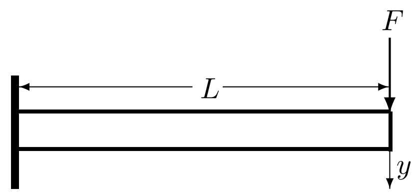

Figure 9: Cantilever for problem 53.

53. **Cantilever–2.** (p.228) Consider the cantilever in fig. 9. The force F is applied perpendicular to the elastic beam of length L which is rigidly constrained at the base. The bending stiffness of the beam is EI and the end deflection is $y = FL^3/(3EI)$.

(a) The anticipated force is $\widetilde{F}$, which is positive. The uncertainty in the true force, F, is represented by the info-gap model:

$$\mathcal{U}(h) = \left\{ F : \left| \frac{F - \widetilde{F}}{\sigma} \right| \le h \right\}, \quad h \ge 0 \tag{172}$$

where σ is known and positive. The performance requirement is that the end deflection be no less than the critical value y_c. Derive an explicit expression for the robustness to uncertainty.

(b) Continue part (a) and compare two designs with different bending stiffnesses and load uncertainties:

$$(EI)_1 > (EI)_2 \quad \text{and} \quad \sigma_1 < \sigma_2 \tag{173}$$

For what values of critical deflection, y_c, is design $[(EI)_1, \sigma_1]$ preferred over design $[(EI)_2, \sigma_2]$?

(c) Now consider a different performance requirement: the bending moment at the base of the beam must not exceed the critical value M_c. Use the info-gap model of eq.(172) to derive an explicit expression for the robustness to uncertainty.

(d) Derive an expression, based on parts (a) and (c), for the robustness to uncertainty when both of the performance requirements must be satisfied.

(e) Let F be a non-negative random variable with probability density function (pdf) $p(F)$ whose estimated form is exponential: $\widetilde{p}(F) = \lambda e^{-\lambda F}$. The uncertainty in the pdf is represented by:

$$\mathcal{U}(h) = \left\{ p(F) : \ p(F) \ge 0, \ \int_0^\infty p(F)\, dF = 1, \ |p(F) - \widetilde{p}(F)| \le h\widetilde{p}(F) \right\}, \quad h \ge 0 \tag{174}$$

The mechanical system fails if the deflection, y, is less than y_c. The performance requirement is that the probability of failure must not exceed P_c. Derive an explicit expression for the robustness of this performance function, for P_c much less than 1.

(f) Now suppose that N forces, $f = (f_1, \ldots, f_N)$, are applied perpendicularly to the beam, where f_i is applied at a distance ℓ_i from the base. As in part (c), the performance requirement is that the bending moment at the base of the beam must not exceed the critical value M_c. The nominal force vector is $\widetilde{f}$, and uncertainty is represented as:

$$\mathcal{U}(h) = \left\{ f : \ (f - \widetilde{f})^T W (f - \widetilde{f}) \le h^2 \right\}, \quad h \ge 0 \tag{175}$$

where W is a known, positive definite, symmetric matrix. Derive an explicit expression for the robustness.

(g) Return to part (a) and denote the robustness $\widehat{h}_y$. Suppose that the horizon of uncertainty, h, is a random variable with exponential distribution: $p(h) = \lambda \mathrm{e}^{-\lambda h}$. The system fails if the end deflection is less than y_{c}. Derive an upper bound for the probability of failure, as a function of $\widehat{h}_y$. This upper bound is less than one.

(h) Consider the end-loaded beam in fig. 9, where $L = 1$m and $F = 1000$N. The end deflection was measured 5 times with normal noise, and the observed deflections are 0.016, 0.010, 0.013, 0.011 and 0.012m. Use a statistical test to decide between the following two hypotheses:

$$H_0 : \qquad EI = 2 \times 10^4 \text{Nm}^2 \tag{176}$$

$$H_1 : \qquad EI > 2 \times 10^4 \text{Nm}^2 \tag{177}$$

Do you reject H_0 at 0.05 level of significance?

(i) The beam in fig. 9 is loaded repeatedly and the deflection is measured and categorized as “low”, “medium” or “high”. Under normal conditions the probabilities of these categories are:

$$p_{\text{low}} = 0.35, \; p_{\text{med}} = 0.55, \; p_{\text{high}} = 0.10 \tag{178}$$

In the last batch of loadings the observations are:

$$n_{\text{low}} = 55, \; n_{\text{med}} = 75, \; n_{\text{high}} = 20 \tag{179}$$

The null hypothesis is that the conditions are normal. Do you reject the null hypothesis at 0.05 level of significance?