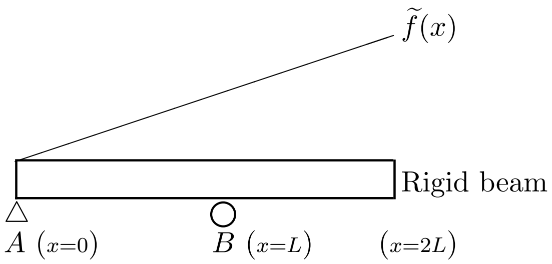

Figure 7: Rigid beam for problem 41.

41. **Trigger mechanism.** (p.89) Consider a completely rigid beam of length $2L$ as shown in fig. 7, with simple supports A and B at points $x = 0$ and $x = L$. The distributed load acts perpendicularly to the beam, with positive force directed downward. The estimated load is:

$$\widetilde{f}(x) = \mu x/L \tag{122}$$

where $\mu > 0$.

The uncertainty in the load is represented by:

$$\mathcal{U}(h) = \left\{ f(x) : \left| \frac{f(x) - \widetilde{f}(x)}{\mu} \right| \leq h \right\}, \quad h \geq 0 \tag{123}$$

We require that the reaction force at support B be no less than the critical value $R_{\rm c}$.

Derive an explicit expression for the robustness function.

Solution for problem 41. (p.23) Let $R_B(f)$ denote the support reaction at point B given distributed force $f(x)$. From a free body diagram we find that the moment of force at support A, which must equal zero, satisfies:

$$M_A = 0 = R_B(f)L - \int_0^{2L} x f(x)\,\mathrm{d}x \tag{731}$$

Hence:

$$R_B(f) = \frac{1}{L}\int_0^{2L} x f(x)\,\mathrm{d}x \tag{732}$$

The robustness is defined as:

$$\widehat{h} = \max\left\{ h : \left(\min_{f\in\mathcal{U}(h)} R_B(f) \right) \geq R_\mathrm{c} \right\} \tag{733}$$

Denote the minimum in eq.(733) by $\phi(h)$. The minimum occur when $f(x) = \widetilde{f}(x) - h\mu$. Thus:

$$\phi(h) = \frac{1}{L}\int_0^{2L} \left(\frac{x}{L} - h\right) x\mu\,\mathrm{d}x = \left(\frac{8}{3} - 2h\right)\mu L \tag{734}$$

From this we find the robustness to be:

$$\widehat{h} = \begin{cases} 0 & \text{if } \frac{8\mu L}{3} < R_\mathrm{c} \\ \frac{4}{3} - \frac{R_\mathrm{c}}{2\mu L} & \text{else} \end{cases} \tag{735}$$

Note crossing of robustness curves: As μL gets larger the curve moves to the right and becomes flatter. However, this crossing occurs at $R_\mathrm{c} = 0$.