103. **Multi-site adverse events** (based on exam in 035018, 26.9.2019), (p.360). Adverse events (fires, earthquakes, terror attacks, etc.) can occur simultaneously at N distinct sites (residential, industrial, commercial, etc.). Each site requires teams made up of a combination of various resources (fire fighters, policemen, psychologists, etc.). Historical data indicate that, on average, the number of teams required at site i is $\overline{q}_i$, for $i = 1, \ldots, N$. However, the actual average requirement at site i, q_i, is uncertain. You work in the emergency-response planning office. You are responsible for choosing the average number of teams, $q_i^\star$, that should be allocated to site i, for $i = 1, \ldots, N$. The uncertainty in the vector, q, of average allocation requirements is represented by this fractional-error info-gap model:

$$\mathcal{U}(h) = \left\{ q : \; q_i \geq 0, \; \left| \frac{q_i - \overline{q}_i}{s_i} \right| h, \; i = 1, \ldots, N \right\}, \quad h \geq 0 \tag{528}$$

where the $\overline{q}_i$'s and s_i's are known positive values.

(a) The performance requirement is:

$$\sum_{i=1}^{N} (q_i^\star - q_i) w_i \geq \delta \tag{529}$$

where the w_i's are known positive values and δ is a specified safety margin. Derive an explicit algebraic expression for the robustness to uncertainty in the required average allocation.

(b) Consider a different allocation requirement:

$$q_i^\star - q_i \geq \delta \quad \text{for each } i = 1, \ldots, N \tag{530}$$

where δ is a specified safety margin. Derive an explicit algebraic expression for the robustness to uncertainty in the required average allocation.

(c) Consider a different info-gap model for uncertainty:

$$\mathcal{U}(h) = \left\{ q : \; (q - \overline{q})^T \, V \, (q - \overline{q}) \leq h^2 \right\}, \quad h \geq 0 \tag{531}$$

where V is a known, symmetric, real, positive definite matrix. Using the performance requirement in eq.(529), derive an explicit algebraic expression for the robustness to uncertainty in the required average allocation.

(d) ‡ Repeat part 103a with a single change. The w_i are known values, but some are positive (penalty for under-allocation) and some are negative (penalty for over-allocation). Derive an explicit algebraic expression for the robustness to uncertainty in the required average allocation.

(e) We now introduce a loss function, $\ell(q_i^\star)$, which represents the loss at site i from average allocation $q_i^\star$. We require that the total loss be no greater than a critical value, ℓ_{c}:

$$\sum_{i=1}^{N} \ell(q_i^\star) \leq \ell_{\mathrm{c}} \tag{532}$$

Furthermore, there are two different policy options. For example, option 1 has more police but fewer psychologists, while option 2 is the reverse. However, the loss function for each policy is uncertain, according to this info-gap model for policy option j:

$$\mathcal{U}_j(h) = \left\{ \ell(q_i^\star) : \; \ell(q_i^\star) \geq 0, \; \left| \frac{\ell(q_i^\star) - \widetilde{\ell}_j(q_i^\star)}{v_j \widetilde{\ell}_j(q_i^\star)} \right| \leq h \right\}, \quad h \geq 0, \quad j = 1, 2 \tag{533}$$

where $\widetilde{\ell}_j(q_i^\star)$ and v_j are known and positive. Also:

$$\sum_{i=1}^{N} \widetilde{\ell}_1(q_i^\star) \quad < \quad \sum_{i=1}^{N} \widetilde{\ell}_2(q_i^\star) \tag{534}$$

$$v_1 \sum_{i=1}^{N} \widetilde{\ell}_1(q_i^\star) \quad > \quad v_2 \sum_{i=1}^{N} \widetilde{\ell}_2(q_i^\star) \tag{535}$$

For what values of $\ell_{\rm c}$ is option 2 preferred, according to the method of robust-satisficing?

(f) The severity of fires is measured with a non-negative scalar variable s. For residential fires the probability density function (pdf) for s is exponential:

$$p(s) = \lambda {\rm e}^{-\lambda s}, \quad s \geq 0 \tag{536}$$

where $\lambda = 0.01$. For non-residential fires (e.g. industrial fires) s tends to take larger values than for residential fires. A specific fire had an observed positive value of severity, $s_{\rm o} = 500$. Formulate an explicit algebraic expression for the level of statistical significance to decide between the following two hypotheses regarding this fire:

$$H_0 : \qquad \text{Residential fire} \tag{537}$$

$$H_1 : \qquad \text{Non-residential fire} \tag{538}$$

Do you accept H_0 at the 0.025 level of significance?