33. **Spatial monitoring, simple.** (p.202) The density $\rho(x)$ of some highly undesirable material (e.g., toxin, invasive species, chemical impurity, etc.) varies along a transect from $x = 0$ to $x = L$. The true value of the total quantity is $r(\rho) = \int_0^L \rho(x)\,\mathrm{d}x$. If r exceeds a very small critical amount, r_{c}, then remedial action will be taken. You will perform N measurements to verify that *none* of this material is present at positions $x_1, \ldots, x_N$. The density tends to be constant along the transect, but the actual slope of the density varies by an unknown amount along the transect. Let $\rho'(x)$ denote the derivative of the density function. Given the measurements, the following info-gap model represents the spatial uncertainty in the true density function:

$$\mathcal{U}(h) = \left\{\rho(x) : \ \rho(x_i) = 0,\ i = 1, \ldots, N,\ \left|\rho'(x)\right| \le h\right\}, \quad h \ge 0 \tag{112}$$

(a) Suppose you perform two measurements, one at each end of the interval. Formulate and evaluate the robustness to spatial uncertainty.

(b) Suppose you perform $N + 1$ evenly spaced measurements, including one at each end of the interval. Formulate and evaluate the robustness to spatial uncertainty.

34. **Spatial monitoring.** (p.202) The density $\rho(x)$ of some material of interest (e.g., rare plants, valuable minerals, chemical impurity, seismic faults, etc.) varies along a transect from $x = 0$ to $x = 1$. You will perform N measurements, obtaining the results $m_i = \rho(x_i)$, $i = 1, \ldots, N$. Your estimate of the mean density is $\overline{m} = (1/N)\sum_{i=1}^N m_i$. The true value of the average density is $\mu = \int_0^1 \rho(x)\,\mathrm{d}x$. The density tends to be constant, but the actual slope of the density varies by an unknown amount along the transect. Let $\rho'(x)$ denote the derivative of the density function. Given the measurements, the following info-gap model represents the spatial uncertainty in the true density function:

$$\mathcal{U}(h) = \left\{\rho(x) : \ \rho(x_i) = m_i,\ i = 1, \ldots, N,\ \left|\rho'(x)\right| \le h\right\}, \quad h \ge 0 \tag{113}$$

You require that the absolute difference between the estimate, $\overline{m}$, and the true value, μ, be no greater than ε.

(a) Suppose you perform a single measurement at the midpoint, $x_1 = 1/2$. Formulate and evaluate the robustness to spatial uncertainty.

(b) Suppose you perform two measurements, one at each end of the interval. Formulate and evaluate the robustness to spatial uncertainty.