Lecture 3

Probabilistic Reliability

with

Info-Gap Uncertainty

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1 *Reliability Assessment with Info-Gaps*

1.1 *Introduction*

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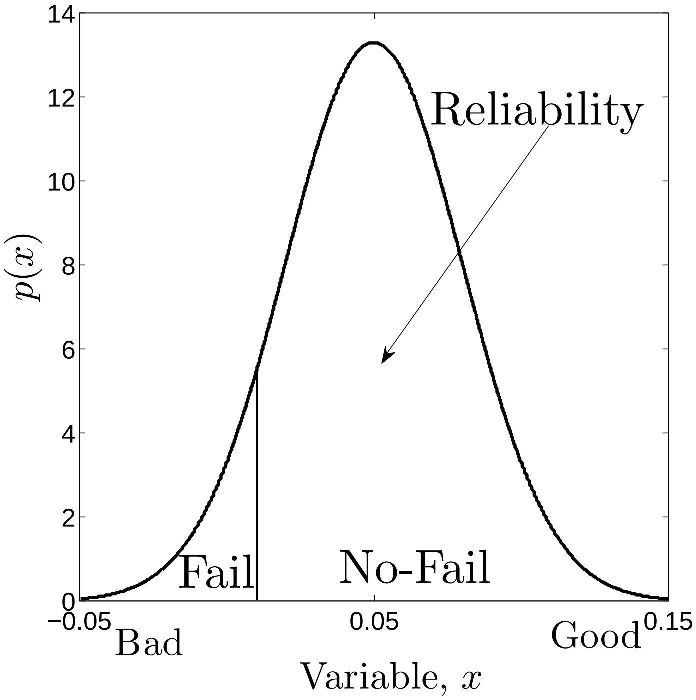
- Robustify against uncertainty.

§ Methodology: Info-gap decision theory.

1.2 *The Problem*



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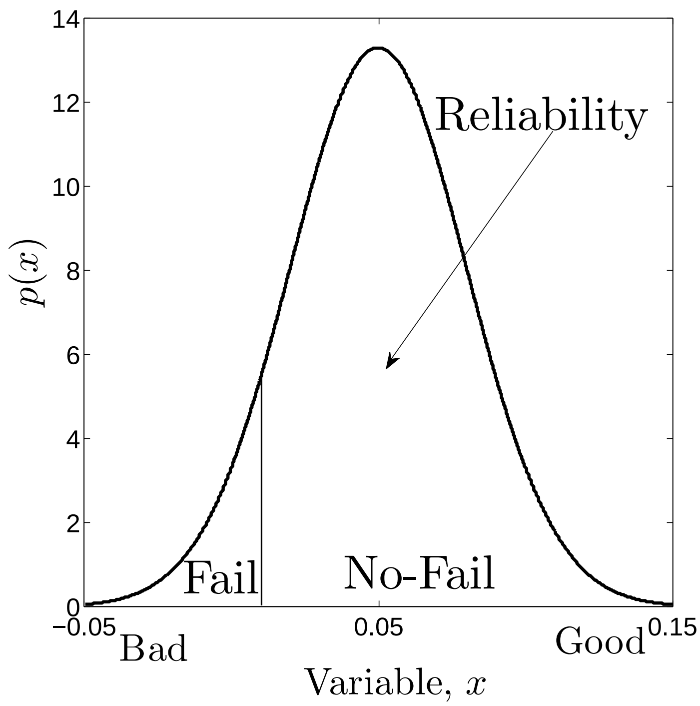

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§ **Good news: "Fail" area usually very small.**

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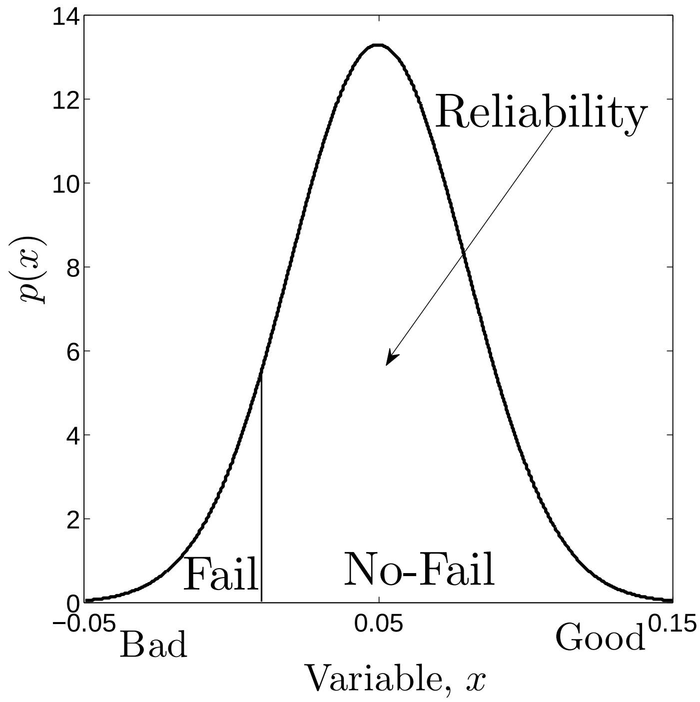

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- **Sensitive to error in probability function.**

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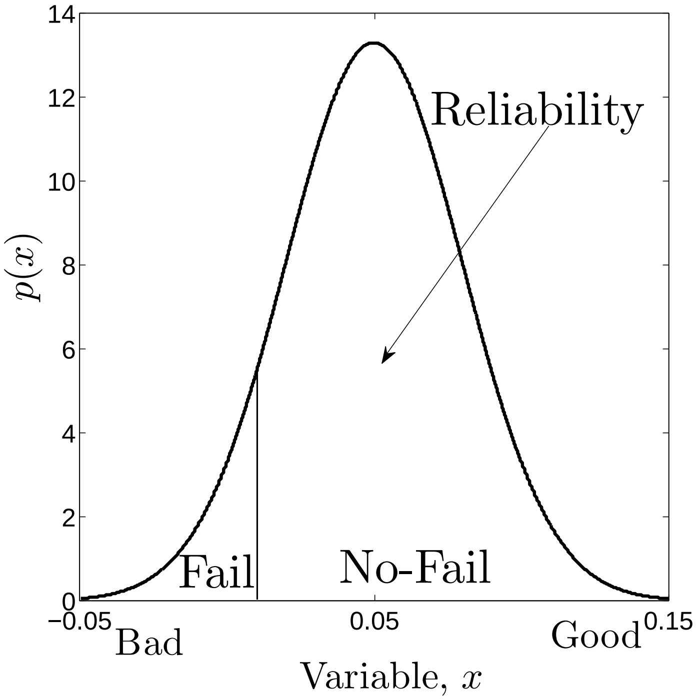

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§ **Good news: Info-gap theory.**

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  - Data are revised.
  - Shackle-Popper indeterminism.
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- **Fat tails of the probability distribution:**
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- **Past vs future:**
  - **Processes vary in time.**
  - **Data are revised.**
  - **Shackle-Popper indeterminism.**
- **Joint probabilities:**
  - **Uncertain common-mode failures.**
  - **Uncertain correlations.**

§ Two foci of uncertainty:

- **Statistical fluctuations:**
  - **Randomness, “noise”.**
  - **Estimation uncertainty.**
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- **Statistical fluctuations:**
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- **Knightian uncertainty:**
  - **Surprises.**
  - **Structural changes.**
  - **Historical data used to predict future.**

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§ **Info-gap theory to manage**

Knightian uncertainty.

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2 *Zoonotic Disease*

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- **Distance of disease front from sea seems to be:**

$$x(t) = a\sqrt{t} \tag{2}$$

- **Our town located distance x_c from the sea.**
- **Estimated time of arrival:**

$$t_c = \left(\frac{x_c}{a}\right)^2 \tag{3}$$

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§ New zoonotic disease:

- **Moving up an estuary from the sea.**
- **Distance of disease front from sea seems to be:**

$$x(t) = a\sqrt{t} \tag{4}$$

- **Our town located distance $x_{\rm c}$ from the sea.**
- **Estimated time of arrival:**

$$t_{\rm c} = \left(\frac{x_{\rm c}}{a}\right)^2 \tag{5}$$

§ The problem: eq.(1) highly uncertain.

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- **Moving up an estuary from the sea.**
- **Distance of disease front from sea seems to be:**

$$x(t) = a\sqrt{t} \tag{6}$$

- **Our town located distance $x_{\rm c}$ from the sea.**
- **Estimated time of arrival:**

$$t_{\rm c} = \left(\frac{x_{\rm c}}{a}\right)^2 \tag{7}$$

§ The problem: eq.(6) highly uncertain.

- **We require time T for intervention.**
- **Probability of failure:** $P_{\rm f}(T) = {\rm Prob}(t_{\rm c} < T)$.

§ $p(a)$ = PDF for a:

$$p(a) = \lambda \mathrm{e}^{-\lambda a}, \quad a \geq 0 \tag{8}$$

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§ $p(a)$ = **PDF for** a**:**

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§ **Info-gap model of uncertain** λ**:**

$$\mathcal{U}(h) = \left\{ \lambda : \ \lambda \geq 0, \ \left| \frac{\lambda - \widetilde{\lambda}}{s} \right| \leq h \right\}, \quad h \geq 0 \tag{12}$$

- ◦ **Non-probabilistic uncertainty.**
- ◦ **No known worst case.**

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§ **Robustness: maximum tolerable uncertainty.**

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Maximum horizon of uncertainty in λ with acceptable probability of failure:

$$\widehat{h}(T) = \max \left\{ h : \ \left(\max_{\lambda \in \mathcal{U}(h)} P_{\rm f}(T) \right) \le P_{\rm c} \right\} \qquad (15)$$

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- **Recall info-gap model of uncertainty:**

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§ **Robust satisficing:**

- **Satisfice performance and**
- **Maximize robustness to uncertainty.**
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§ **Robust satisficing:**

- **Satisfice performance and**
- **Maximize robustness to uncertainty.**
- **Not min-max (minimizing a worst case).**
- **Not putative outcome optimization:** $\min P_{\mathrm{f}}$.

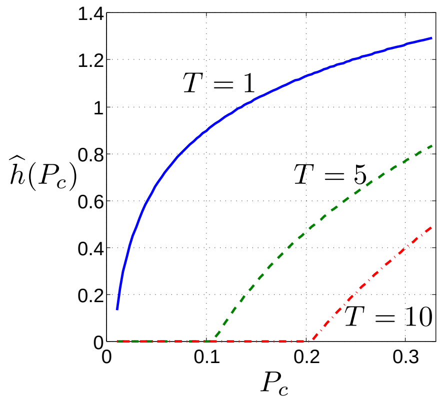

Figure 1: Robustness curves with $T = 1$ (solid), $T = 5$ (dash) and $T = 10$ (dot-dash) with parameter values: $\widetilde{\lambda} = 1/2$, $s = 0.3$, $x_c = 10$.

§ Robustness function:

$$\widehat{h}(T) = \frac{1}{s}\left(\widetilde{\lambda} + \frac{\sqrt{T}}{x_c}\ln P_c\right) \tag{22}$$

- **Trade off:** P_c **up (bad)** $\Longleftrightarrow$ $\widehat{h}$ **up (good).**
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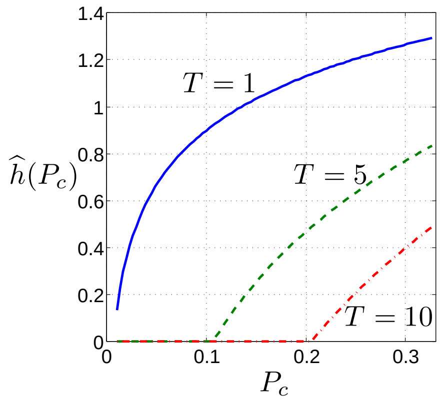

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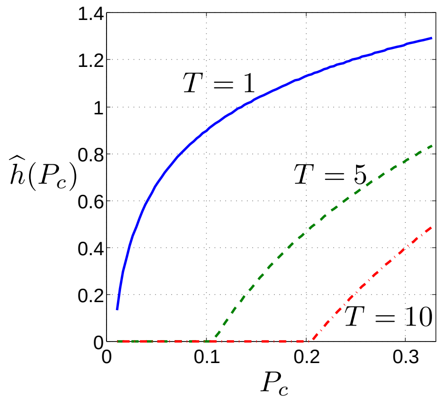

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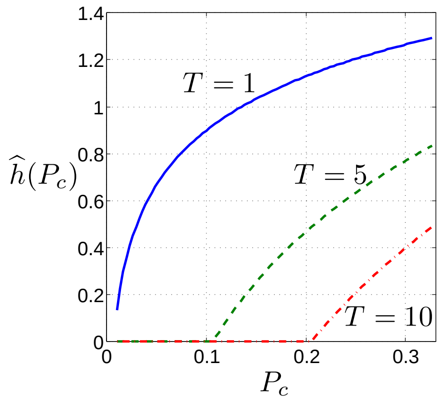
§ Robustness function:

$$\widehat{h}(T) = \frac{1}{s}\left(\widetilde{\lambda} + \frac{\sqrt{T}}{x_{\rm c}} \ln P_{\rm c}\right) \tag{24}$$

- **Trade off:** $P_{\rm c}$ **up (bad)** $\Longleftrightarrow$ $\widehat{h}$ **up (good).**
- **Zeroing: No robustness at predicted** $P_{\rm f}$**.**
- $\widehat{h}$ **up as required intervention time,** T**, reduced.**



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§ What do the robustness numbers mean?

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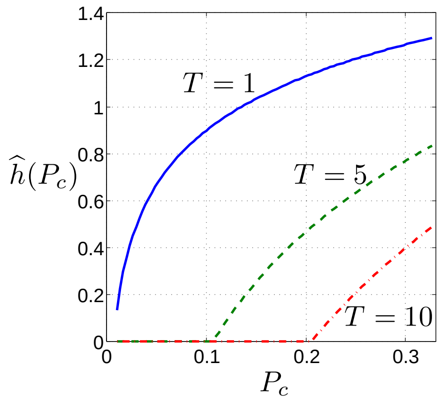

Figure 5: Robustness curves with $T = 1$ (solid), $T = 5$ (dash) and $T = 10$ (dot-dash) with parameter values: $\widetilde{\lambda} = 1/2$, $s = 0.3$, $x_c = 10$.

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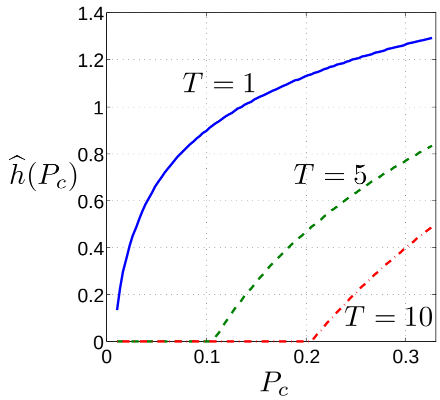
- **Example:** $(P_c, \widehat{h}, T) = (0.2, 0.45, 5)$.
-



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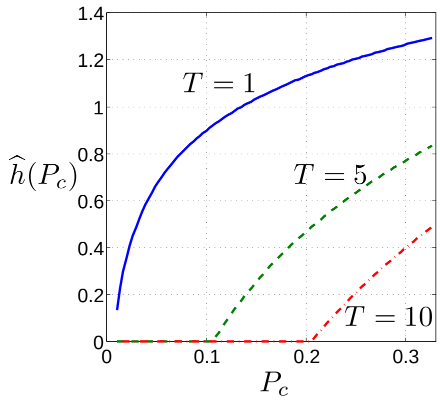

Figure 7: Robustness curves with $T=1$ (solid), $T=5$ (dash) and $T=10$ (dot-dash) with parameter values: $\widetilde{\lambda}=1/2$, $s=0.3$, $x_{\rm c}=10$.

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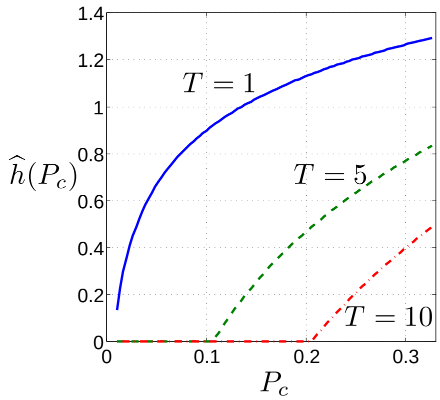

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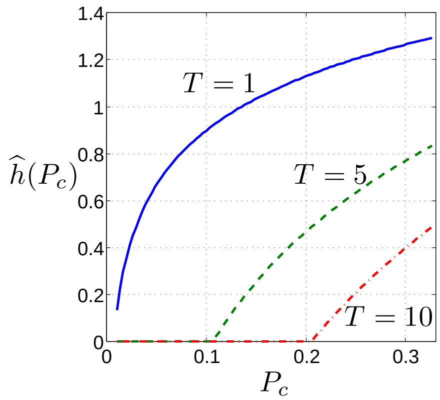

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- **Low robustness for this T and P_c.**
- **Expl: $(P_c, \widehat{h}, T) = (0.2, 1.1, 1)$: moderate robustness.**

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$$\widetilde{\lambda}_1 > \widetilde{\lambda}_2 \tag{25}$$

Recall: $P_{\mathrm{f}}(T) = \exp(-\lambda x_{\mathrm{c}}/\sqrt{T})$.

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- I_1 **predicted to be better than** I_2:

$$\widetilde{\lambda}_1 > \widetilde{\lambda}_2 \tag{26}$$

Recall: $P_{\rm f}(T) = \exp(-\lambda x_{\rm c}/\sqrt{T})$.

- I_1 **more uncertain than** I_2:

$$\frac{s_1}{\widetilde{\lambda}_1} > \frac{s_2}{\widetilde{\lambda}_2} \tag{27}$$

Hence the dilemma.

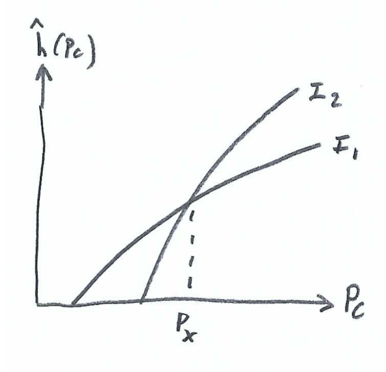

Figure 10: Crossing robustness curves showing preference reversal.

- **Robustness curves cross:**
  Potential for preference reversal.
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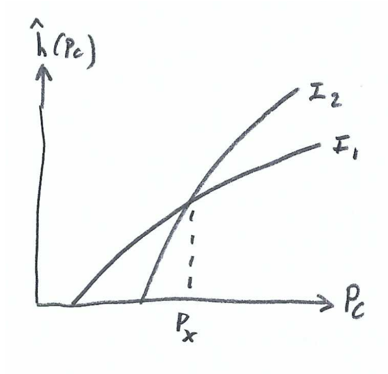

Figure 11: Crossing robustness curves showing preference reversal.

- **Robustness curves cross:**
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- **How to choose? Robust satisficing.**

Satisfice probability of failure. Maximize robustness:

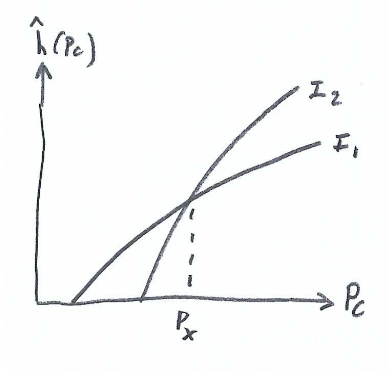

Figure 12: Crossing robustness curves showing preference reversal.

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Satisfice probability of failure. Maximize robustness:

I_1 **preferred if** $P_c < P_\times$.

I_2 **preferred if** $P_c > P_\times$.

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- **Innovation dilemma:**
  - **New & innovative: seems better, more uncertain.**
  - **State of the Art: seems worse, less uncertain.**
- **Resolution: robust satisficing.**

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Any Questions?