Contents

Managing Uncertainty in Decision Making for Conservation Biology

Yakov Ben-Haim
ORCID ID: 0000-0001-7229-8449
Technion — Israel Institute of Technology
Haifa 32000 Israel
yakov@technion.ac.il

1 Online Appendix S1: Formulation and Derivation of Robustness Functions for Section 5

Derivations quite similar to those in this section and the next can be found in Ben-Haim (2006, section 3.2).

1.1 Scalar Performance Function

We consider the selection between several alternative decisions. Let d denote a specific decision and let B denote the set of available decisions. For any decision, d, the decision performance is expressed by a scalar function $M(d)$. However, the form of this function is poorly known. The best known estimate of this function is $\widetilde{M}(d)$, but the accuracy of this estimate is highly uncertain. We assume that the estimate, $\widetilde{M}(d)$, is positive.

We face deep uncertainty regarding the accuracy of the estimated performance function, $\widetilde{M}(d)$. We have no knowledge, probabilistic or otherwise, about how greatly $\widetilde{M}(d)$ errs. The absolute fractional error of the estimated function is $\left|\frac{M(d)-\widetilde{M}(d)}{\widetilde{M}(d)}\right|$, but the value of this fractional error is unknown. It may be small, or large, but we have no information about its value or how this error varies as the decision, d, is changed. Our uncertainty about the estimated performance function is non-probabilistic and unbounded. We represent this deep uncertainty about the estimated performance function of decision d with the following fractional-error info-gap model of uncertainty:

$$\mathcal{U}(h,b)=\left\{M(d):\ \left|\frac{M(d)-\widetilde{M}(d)}{\widetilde{M}(d)}\right|\le h\right\},\quad h\ge 0 \tag{1}$$

For any value of h, the set $\mathcal{U}(h,b)$ contains all performance functions for which the absolute fractional error of the estimated performance is never greater than h. However, the value of h is unknown and unbounded because there is no known worst case or greatest possible error of the

\papers\mil-prepared2020\pol-sel014appendices.tex. Derived from pol-sel013appendices.tex by removing the appendices to a separate file. 7.6.2023 estimated function. The info-gap model of uncertainty is thus not a single set, but rather an unbounded family of nested sets of possible performance functions for decision d. The parameter h has the meaning of an *horizon of uncertainty* because the range of possible performance functions increases as h increases. The info-gap uncertainty is non-probabilistic because no probability functions are involved. Recall that we are assuming that the estimated performance function, $\widetilde{M}(d)$, is positive, though this is not assumed of $M(d)$.

We first consider effectiveness (not cost) as the performance function, for which we would prefer a greater value over a lesser value. When facing deep uncertainty it is wise to ask: What is the least effectiveness that would be acceptable, or equivalently, how good is good enough, or how bad is still acceptable? These questions focus on satisficing the effectiveness, rather than optimizing it.

We express the satisficing requirement by demanding that the actual effectiveness of a decision that we are considering, $M(d)$, be no less than a critical value, M_{c}:

$$M(d) \geq M_{\mathrm{c}} \tag{2}$$

The critical effectiveness, M_{c}, is a parameter, and we explore the feasibility and reliability of satisfying the requirement in eq.(2), as M_{c} varies, in light of the uncertainty about $M(d)$.

Because the true effectiveness function, $M(d)$, is unknown, we cannot know whether or not a proposed decision, d, is good enough according to eq.(2). However, we can evaluate the degree of robustness to uncertainty of a proposed decision. That is, if the estimated effectiveness, $\widetilde{M}(d)$, exceeds M_{c}, then we ask: how much error in the function $\widetilde{M}(d)$ can we tolerate, and the correct function, $M(d)$, still satisfies the requirement in eq.(2) with decision d? The answer to this question is the robustness of decision d. If the robustness is large then we have high confidence that this decision will satisfy the requirement; small robustness implies low confidence.

The robustness function for decision d with required effectiveness M_{c} is defined as:

$$\widehat{h}(d, M_{\mathrm{c}}) = \max \left\{ h : \ \left(\min_{M \in \mathcal{U}(h,b)} M(d) \right) \geq M_{\mathrm{c}} \right\} \tag{3}$$

Reading this equation from left to right: the robustness, $\widehat{h}(d, M_{\mathrm{c}})$, of decision d with critical effectiveness M_{c}, is the maximum horizon of uncertainty, h, up to which all realizations of the effectiveness function, $M(d)$, are acceptable.

Let $s(h)$ denote the inner minimum in the definition of the robustness function, eq.(3). This minimum occurs, for the info-gap model of eq.(1), when $M(d) = (1-h)\widetilde{M}(d)$. Equating this minimum to the critical value, M_{c}, and solving for h yields the robustness function:

$$\widehat{h}(d, M_{\mathrm{c}}) = 1 - \frac{M_{\mathrm{c}}}{\widetilde{M}(d)} \tag{4}$$

or zero if this expression is negative, which would occur if the estimated effectiveness, $\widetilde{M}(d)$, does not satisfy eq.(2).

A knowledge-based strategy can employ any combination of scientific models, empirical relations, or expert judgment. A knowledge-based strategy chooses the decision, d, that maximizes the best estimate of the effectiveness function, $\widetilde{M}(d)$. We denote this knowledge-based selection d_{k}, and it is formally defined as:

$$d_{\mathrm{k}} = \arg\max_{b \in B} \widetilde{M}(d) \tag{5}$$

Thus $\widetilde{M}(d_{\mathrm{k}})$ is no less than $\widetilde{M}(d)$ for any available decision, d.

From eqs.(4) and (5) we see that the knowledge-based selection, $d_{\rm k}$, maximizes the robustness to uncertainty in the decision effectiveness. That is, the robustness of $d_{\rm k}$ is strictly greater than the robustness of any available decision, d, that does not maximize the predicted effectiveness:

$$\widehat{h}(d_{\rm k}, M_{\rm c}) > \widehat{h}(d, M_{\rm c}) \quad \text{for all} \quad 0 < M_{\rm c} < \widetilde{M}(d_{\rm k}) \tag{6}$$

Eq.(6) asserts the robust dominance of the knowledge-based strategy for decision making, over any other strategy, when effectiveness is assessed with a single scalar function and when $\widetilde{M}(d)$ is the best known estimate of the effectiveness.

If the performance function is loss rather than effectiveness then we require that $M(d)$ not exceed the critical value $M_{\rm c}$. Hence the inequalities in eqs.(2) and (3) are reversed and the inner minimum in eq.(3) becomes a maximum. Following analogous reasoning one find the robustness function for loss:

$$\widehat{h}(d, M_{\rm c}) = \frac{M_{\rm c}}{\widetilde{M}(d)} - 1 \tag{7}$$

or zero if this is negative. The robust dominance of the knowledge-based strategy that minimizes the estimated loss holds in this case as well.

Other than assuming that the best known performance function for either effectiveness or loss, $\widetilde{M}(d)$, is a positive scalar function, we have made no assumptions about either the true or the estimated scalar performance functions, $M(d)$ or $\widetilde{M}(d)$, nor about the class of available decisions B. This very general result — that knowledge-based decision making strategies are more robust than all other strategies — holds because of the extreme paucity of information about the scalar performance function as expressed by the info-gap model of eq.(1). This is a somewhat ironic result: extreme lack of knowledge justifies the knowledge-based decision making strategy. From a contrarian perspective one might say that the knowledge-based strategy is better than any other strategy simply because we know so little about the processes involved. We will explore the limits of this conclusion. This ironic result is also a note of caution against overconfidence in knowledge-based optimization of decision performance.

1.2 Scalar Effectiveness Function with an Error Estimate: Innovation Dilemma

We now continue the previous development, but consider further information. In addition to an estimate of the scalar performance function, $\widetilde{M}(d)$, as a function of the decision, d, we also have a positive error estimate, $w(d)$. That is, it is thought that the true performance, $M(d)$, may deviate from $\widetilde{M}(d)$ by as much as $\pm w(d)$ or more. The error function $w(d)$ does not provide an upper bound or worst case of the error, but it does compare the anticipated errors for different decisions d: some decisions have lower estimated error than other decisions.

For instance, $w(d)$ could quantify contextual understanding that suggests that the estimated performance is less uncertain for a decision with large support to human development and small support to physical infrastructure, than the reverse. More generally, the error estimate $w(d)$ can reflect relatively lower uncertainty about the capabilities of specific agents or organizations, as demonstrated by those entities in their past behavior.

We incorporate the additional information provided by the error function $w(d)$ by modifying the info-gap model for uncertainty in the scalar performance, eq.(1), as follows:

$$\mathcal{U}(h, b) = \left\{ M(d) : \left| \frac{M(d) - \widetilde{M}(d)}{w(d)} \right| \leq h \right\}, \quad h \geq 0 \tag{8}$$

This is the same as eq.(1) if $w(d) = \widetilde{M}(d)$.

When the performance is assessed as effectiveness, the performance requirement is the same as eq.(2), and eq.(3) is the definition of the robustness function. Arguing as before, we obtain the following expression for the robustness to uncertainty in the estimated performance function:

$$\widehat{h}(d, M_c) = \frac{\widetilde{M}(d) - M_c}{w(d)} \tag{9}$$

or zero if this expression is negative. This reduces to eq.(4) if $w(d) = \widetilde{M}(d)$.

When the performance is measured as a loss the robustness function becomes:

$$\widehat{h}(d, M_c) = \frac{M_c - \widetilde{M}(d)}{w(d)} \tag{10}$$

or zero if this expression is negative. This reduces to eq.(7) if $w(d) = \widetilde{M}(d)$.

2 Online Appendix S2: Formulation and Derivation of the Robustness Function for Section 6

$$\widehat{h}(b_c) = \max \left\{ h : \left(\min_{T_f, b \in \mathcal{U}(h)} b(T_f) \right) \geq b_c \right\} \tag{11}$$

Let $m(h)$ denote the inner minimum in eq.(11). This is the inverse function of the robustness function, $\widehat{h}(b_c)$. That is, a plot of h vs. $m(h)$ is identical to a plot of $\widehat{h}(b_c)$ vs. b_c. It is thus sufficient to derive an expression for $m(h)$.

The inner minimum in eq.(11) occurs when the reproductive output function is as low as possible at horizon of uncertainty h. According to the info-gap model, and for $h \leq 1$, this is:

$$b(T_f) = (1 - h)\widetilde{b}(T_f) \tag{12}$$

Because the estimated reproductive output function, $\widetilde{b}(T)$, is unimodal, and still considering $h \leq 1$, the value of T_f that produces the inner minimum is either the minimum or the maximum value: $(1 - h)\widetilde{T}_f$ or $(1 + h)\widetilde{T}_f$. Thus the inner minimum in eq.(11) is the lesser of the following two alternatives:

$$m_1(h) = (1 - h)\widetilde{b}\left((1 - h)\widetilde{T}_f\right) \tag{13}$$

$$m_2(h) = (1 - h)\widetilde{b}\left((1 + h)\widetilde{T}_f\right) \tag{14}$$

That is, the inverse of the robustness function, for $h \leq 1$, is:

$$m(h) = \min[m_1(h),\ m_2(h)] \tag{15}$$

Eqs.(13)–(15), together with the specification of $\widetilde{b}(T)$, provide the numerical basis for evaluating and plotting the robustness function, $\widehat{h}(b_c)$.