101. **Energy conservation by feedback** (based on exam in 035018, 22.5.2019), (p.352). People change their energy consumption in response to feedback about their prior energy use. Define:

$n(c)\,\mathrm{d}c =$ number of consumers whose prior energy consumption was in the interval $[c,\ c+\mathrm{d}c]$.

The estimated consumption in the next time interval, for a consumer whose prior consumption was in the interval $[c,\ c+\mathrm{d}c]$, is denoted $\widetilde{f}(c,\rho)$, where ρ is a parameter expressing the intensity of the feedback; greater ρ implies greater intensity.

The true consumption function is $f(c,\rho)$, whose uncertainty is represented by an info-gap model, $\mathcal{U}(h)$. The response of the entire population to feedback at intensity ρ is:

$$R(\rho, f) = \int_0^{\infty} f(c,\rho) n(c)\,\mathrm{d}c \tag{505}$$

We require that the population response be no greater than the critical value, R_{c}:

$$R(\rho, f) \le R_{\mathrm{c}} \tag{506}$$

(a) Derive an explicit algebraic expression for the robustness function for the following fractional-error info-gap model:

$$\mathcal{U}(h) = \left\{ f(c,\rho):\ f(c,\rho) \ge 0,\ \left| \frac{f(c,\rho) - \widetilde{f}(c,\rho)}{\widetilde{f}(c,\rho)} \right| \le h \right\},\quad h \ge 0 \tag{507}$$

(b) Derive an explicit algebraic expression for the robustness function for the following fractional-error info-gap model:

$$\mathcal{U}(h) = \left\{ f(c,\rho):\ f(c,\rho) \ge 0,\ \left| \frac{f(c,\rho) - \widetilde{f}(c,\rho)}{w} \right| \le h \right\},\quad h \ge 0 \tag{508}$$

where w is a known positive constant.

(c) Continuing from part 101b, consider two different situations, $(\rho_1,\ w_1)$ and $(\rho_2,\ w_2)$, where:

$$\rho_2 < \rho_1 \quad \text{and} \quad 0 < w_2 < w_1 \tag{509}$$

That is, the feedback in situation 1 is more intensive, but the uncertainty in this situation is greater. For what values of R_{c} is situation 2 robust-preferred? Assume that $\widetilde{f}(c,\rho) = (1-\rho)c$.

(d) For a particular info-gap model, the robustness function takes this form:

$$\widehat{h}(R_{\mathrm{c}}, \rho) = (R_{\mathrm{c}} - w)\rho \tag{510}$$

or zero if this is negative, where ρ and w are positive constants. Consider two different situations, $(\rho_1,\ w_1)$ and $(\rho_2,\ w_2)$, where:

$$0 < \rho_2 < \rho_1 \quad \text{and} \quad 0 < w_2 < w_1 \tag{511}$$

For what values of R_{c} is situation 1 robust-preferred?

(e) The true and estimated consumption functions are related as:

$$f(c,\rho) = \widetilde{f}(c,\rho) + \sum_{j=1}^{J} a_j \sin \frac{j\pi c}{c_{\max}} \tag{512}$$

$$= \widetilde{f}(c,\rho) + a^T \sigma(c) \tag{513}$$

where $c_{\rm max}$ is a known positive number, and a and $\sigma(c)$ are the vectors of Fourier coefficients and sine functions in eq.(512). The uncertainty in $f(c,\rho)$ is represented by this Fourier-ellipsoid info-gap model:

$$\mathcal{U}(h) = \left\{ f(c,\rho) = \widetilde{f}(c,\rho) + a^T\sigma(c) : \ a^T W a \le h^2 \right\}, \quad h \ge 0 \tag{514}$$

where W is a known, positive definite, real, symmetric matrix. Derive an explicit algebraic expression for the robustness function.