Lecture 2

Info-Gap Robustness of a Beam

with

Uncertain Load

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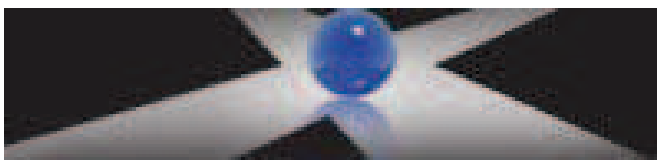

Contents

1 *Info-Gap Robustness of a Beam With an Uncertain Load*

(Source: Yakov Ben-Haim, 1996, *Robust Reliability in the Mechanical Sciences*, Springer, sections 3.1, 3.2.)

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- **Uncertainty in functional shape, not just parameters.**
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§ We wish to

- **Analyze and enhance reliability.**
- **Evaluate different levels and types of information.**

§ What we do know about the load:

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- **The precise realization of the load density,** $\phi(x)$**.**
- **The bound on the deviation of** $\phi(x)$ **from** $\widetilde{\phi}(x)$**.**

§ **The disparity between what we do know and what we need to know for a fully competent design or analysis is an information gap.**

§ Info-gap model of load uncertainty:

$$\mathcal{U}(h) = \left\{ \phi(x) : \ \left| \phi(x) - \widetilde{\phi}(x) \right| \le h \right\}, \quad h \ge 0 \qquad (1)$$

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§ Two levels of uncertainty in an info-gap model:

- **At fixed h: true load profile $\phi(x)$ is unknown.**
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§ Two levels of uncertainty in an info-gap model:

- **At fixed h: true load profile $\phi(x)$ is unknown.**
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§ 2 properties of all info-gap models:

- **Contraction:**

$$\mathcal{U}(0) = \left\{ \widetilde{\phi}(x) \right\} \tag{5}$$

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§ Info-gap model of load uncertainty:

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§ Two levels of uncertainty in an info-gap model:

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- **Horizon of uncertainty — h — is unknown.**

§ 2 properties of all info-gap models:

- **Contraction:**

$$\mathcal{U}(0) = \left\{ \widetilde{\phi}(x) \right\} \tag{7}$$

- **Nesting:**

$$h < h' \quad \Longrightarrow \quad \mathcal{U}(h) \subseteq \mathcal{U}(h') \tag{8}$$

§ System model:

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- **For simple-simple beam one finds:**

$$M(x) = -\frac{L-x}{L}\int_0^x \phi(u)u\,\mathrm{d}u - \frac{x}{L}\int_x^L \phi(u)(L-u)\,\mathrm{d}u \tag{9}$$

where L is the length of the beam.

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where L is the length of the beam.

§ Failure criterion:

If bending moment $M(x)$ exceeds the critical value M_c:

$$\max_{0\le x\le L} |M(x)| > M_\mathrm{c} \tag{11}$$

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  does not violate the failure criterion
  for any load profile up to uncertainty h.
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§ **Robustness, $\widehat{h}$, combines**

- **System model, uncertainty model, failure criterion.**
- **The robustness is maximum tolerable uncertainty:**
  Greatest info-gap, h, such that the system model
  does not violate the failure criterion
  for any load profile up to uncertainty h.
- **We can express robustness, $\widehat{h}$, as:**

$$\widehat{h} = \textbf{maximum tolerable uncertainty} \tag{12}$$

$$= \max\{h : \ \textbf{failure cannot occur}\} \tag{13}$$

$$= \max\left\{h : \ \left(\max_{0\le x\le L} |M(x)|\right) \le M_{\rm c} \ \textbf{for all} \ \phi(x) \ \textbf{in} \ \mathcal{U}(h)\right\} \tag{14}$$

$$= \max\left\{h : \ \left(\max_{\phi\in\mathcal{U}(h,\widetilde{\phi})} \ \max_{0\le x\le L} |M(x)|\right) \le M_{\rm c}\right\} \tag{15}$$

We can invert the order of the maxima inside the set.

§ **The robustness is the greatest h at which the maximum bending moment $M(x)$ does not exceed the critical value M_c:**

$$\underbrace{\frac{(h+\widetilde{\phi})L^2}{8}}_{\text{max bending m'nt}} = \underbrace{M_c}_{\text{critical m'nt}} \quad \Longrightarrow \quad \boxed{\widehat{h}(M_c) = \frac{8M_c}{L^2} - \widetilde{\phi}} \tag{16}$$

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§ **Design implications: the robustness, $\widehat{h}$, increases as:**

- **The beam length L decreases.**
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- **The beam length** L **decreases.**
- **The nominal load** $\widetilde{\phi}$ **decreases.**
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§ **The** **robustness** **is the greatest** h

at which the maximum bending moment $M(x)$

does not exceed the critical value $M_{\rm c}$**:**

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§ **Design implications:** **the robustness,** $\widehat{h}$**, increases as:**

- **The beam length** L **decreases.**
- **The nominal load** $\widetilde{\phi}$ **decreases.**
- **The critical bending moment** $M_{\rm c}$ **increases.**

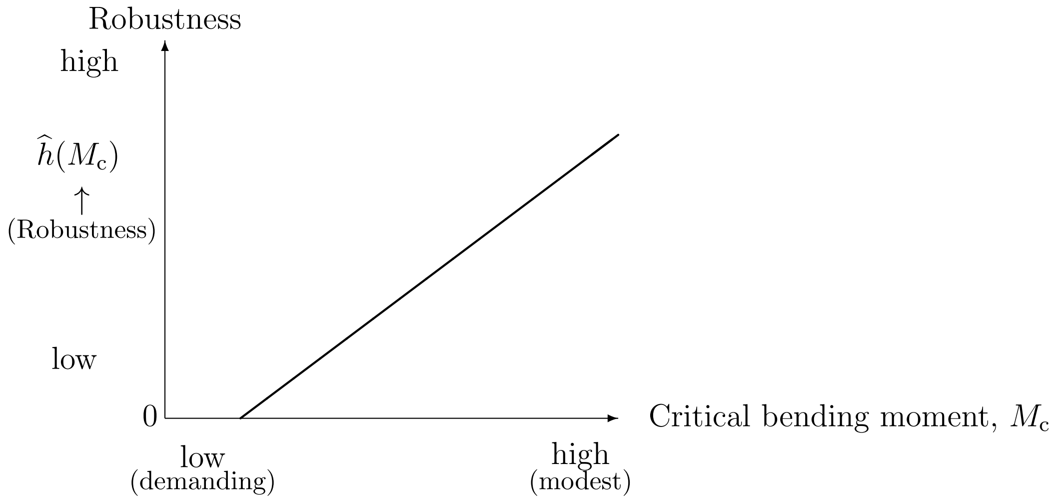

Figure 1: **ROBUSTNESS CURVE.**

§ Two Properties: Trade-off and Zeroing (fig. 1).

§

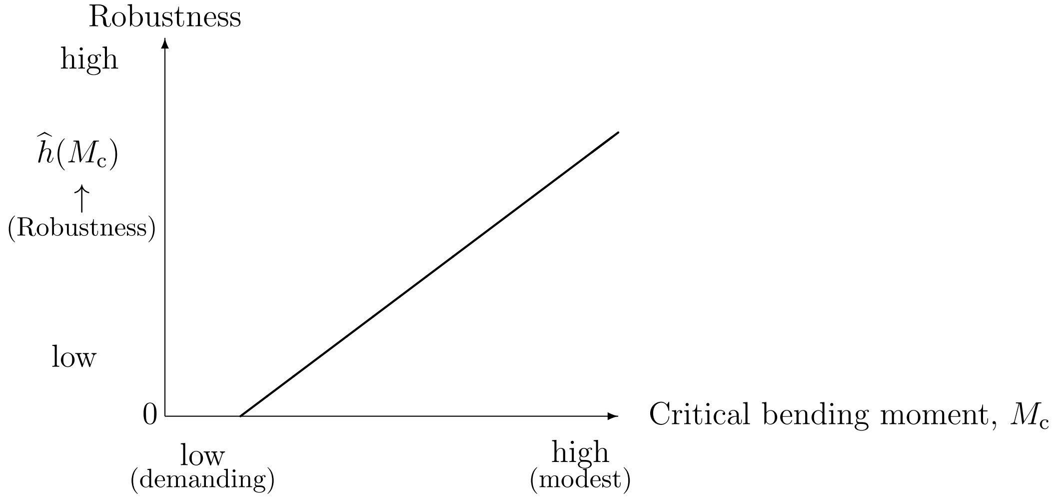

Figure 2: **ROBUSTNESS CURVE.**

§ Two Properties: Trade-off and Zeroing (fig. 2).

§ Trade off: robustness vs performance.

- $\widehat{h}(M_c)$ **gets worse** **(decreases) as** M_c **gets better** **(decreases).**
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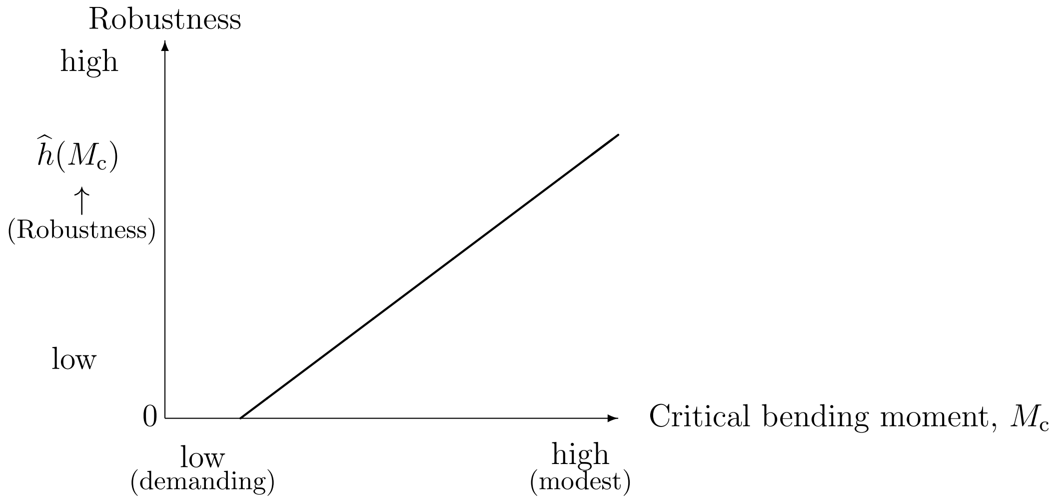

Figure 3: **ROBUSTNESS CURVE.**

§ Two Properties: Trade-off and Zeroing (fig. 3).

§ Trade off: robustness vs performance.

- $\widehat{h}(M_{\rm c})$ **gets worse (decreases) as $M_{\rm c}$ gets better (decreases).**
- **This is the pessimist's theorem.**
-

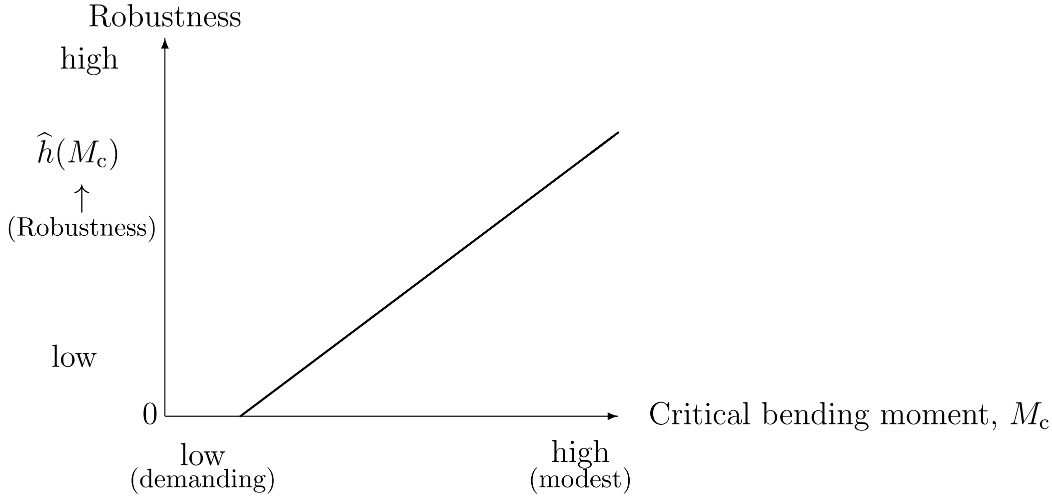

Figure 4: **ROBUSTNESS CURVE.**

§ **Two Properties: Trade-off and Zeroing (fig. 4).**

§ **Trade off: robustness vs performance.**

- $\widehat{h}(M_c)$ **gets worse (decreases) as M_c gets better (decreases).**
- **This is the pessimist's theorem.**
- **Slope of the robustness curve expresses the cost of robustness.**

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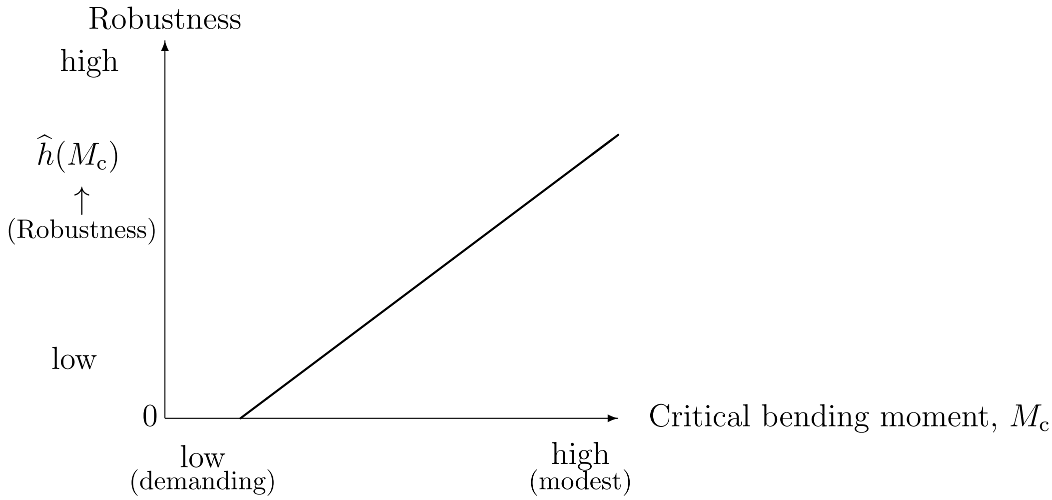

Figure 5: **ROBUSTNESS CURVE.**

§ Two Properties: Trade-off and Zeroing (fig. 5).

§ Trade off: robustness vs performance.

- $\widehat{h}(M_c)$ **gets worse (decreases) as M_c gets better (decreases).**
- **This is the pessimist's theorem.**
- **Slope of the robustness curve expresses the cost of robustness.**

§ Zeroing: Estimated performance has zero robustness:

$$\widehat{h}(M_c) = 0 \quad \textbf{if} \quad M_c = \frac{\widetilde{\phi} L^2}{8} = \textbf{estimated bending moment} \tag{20}$$

2 *Four Info-Gap Models of Uncertainty*

Different prior information:

Different info-gap model of uncertainty.

2.1 *Load-Uncertainty Envelop*

§ **We considered uniform-bound info-gap model, eq.(1):**

$$\mathcal{U}_{\rm uni}(h) = \left\{ \phi(x) : \ \left| \phi(x) - \widetilde{\phi}(x) \right| \le h \right\}, \quad h \ge 0 \qquad (21)$$

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$$\mathcal{U}_{\text{uni}}(h) = \left\{\phi(x): \ \left|\phi(x) - \widetilde{\phi}(x)\right| \le h\right\}, \quad h \ge 0 \qquad (25)$$

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§ **Envelop uncertainty:**

Uncertain deviation of $\phi(x)$ from $\widetilde{\phi}(x)$

varies in an envelop:

$$\mathcal{U}_{\text{env}}(h) = \left\{\phi(x): \ \left|\phi(x) - \widetilde{\phi}(x)\right| \le h\psi(x)\right\}, \quad h \ge 0 \qquad (26)$$

where . . .

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where we know:

- $\widetilde{\phi}(x) =$ **nominal load profile.**
- $\psi(x) =$ **load-uncertainty envelop.**

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where we know:

- $\widetilde{\phi}(x)$ = **nominal load profile.**
- $\psi(x)$ = **load-uncertainty envelop.**

and we do not know:

- $\phi(x)$ = **actual load profile.**
- h = **horizon of uncertainty.**

§ Example: envelop-bound vs. uniform-bound

- **The nominal load increases to the center of the beam:**

$$\widetilde{\phi}(x) = \widetilde{\phi} \sin \frac{\pi x}{L} \tag{31}$$

where $\widetilde{\phi}$ is a known positive constant.

- **The uncertainty increases to the center of the beam:**

$$\psi(x) = \sin \frac{\pi x}{L} \tag{32}$$

so the envelop-bound info-gap model is:

$$\mathcal{U}_{\rm env}(h) = \left\{ \phi(x) : \ \left| \phi(x) - \widetilde{\phi}(x) \right| \le h\psi(x) \right\}, \quad h \ge 0 \tag{33}$$

and the robustness function is:

$$\widehat{h}_{\rm env}(M_{\rm c}) = \frac{\pi^2 M_{\rm c}}{L^2} - \widetilde{\phi} \tag{34}$$

- **Compare to the uniform bound info-gap model:**

$$\mathcal{U}_{\rm uni}(h) = \left\{ \phi(x) : \ \left| \phi(x) - \widetilde{\phi}(x) \right| \le h \right\}, \quad h \ge 0 \tag{35}$$

whose robustness function is:

$$\widehat{h}_{\rm uni}(M_{\rm c}) = \frac{8 M_{\rm c}}{L^2} - \widetilde{\phi} \tag{36}$$

- **Value of information:**

$$\mathcal{U}_{\rm env}(h) \subseteq \mathcal{U}_{\rm uni}(h) \implies \widehat{h}_{\rm env}(M_{\rm c}) \ge \widehat{h}_{\rm uni}(M_{\rm c}) \tag{37}$$

2.2 *Fourier Uncertainty*

§ The info-gap models we considered entail

unbounded rate of variation:

$$\mathcal{U}_{\text{uni}}(h) = \left\{\phi(x) : \left|\phi(x) - \widetilde{\phi}(x)\right| \le h\right\}, \quad h \ge 0 \tag{38}$$

$$\mathcal{U}_{\text{env}}(h) = \left\{\phi(x) : \left|\phi(x) - \widetilde{\phi}(x)\right| \le h\psi(x)\right\}, \quad h \ge 0 \tag{39}$$

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§ **We may have information**

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§ We may have information

constraining the rate of variation

of the uncertain function.

§ For example, frequency-limited function:

$$\phi(x) = \sum_{n=n_1}^{n_2} c_n \cos\frac{n\pi x}{L} \tag{44}$$

$$= c^T \gamma(x) \tag{45}$$

§

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unbounded rate of variation:

$$\mathcal{U}_{\text{uni}}(h) = \left\{\phi(x) : \left|\phi(x) - \widetilde{\phi}(x)\right| \le h\right\}, \quad h \ge 0 \tag{46}$$

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§ Uncertainty in the Fourier coefficients c.

E.g. Fourier ellipsoid-bound info-gap model:

$$\mathcal{U}(h) = \left\{\phi(x) = c^T\gamma(x) : (c - \widetilde{c})^T W (c - \widetilde{c}) \le h^2\right\}, \quad h \ge 0 \tag{50}$$

§ Fourier robustness in a special case ($W = I$):

$$\widehat{h} \approx \frac{n_1^2 \pi^2 M_{\rm c}}{L^2} \tag{51}$$

§ Uniform-bound robustness with $\widetilde{\phi} = 0$:

$$\widehat{h} = \frac{8 M_{\rm c}}{L^2} \tag{52}$$

Reliability is substantially enhanced

by constraining spatial modes of the load function.

2.3 *Energy-Bound Uncertainty*

§ We have considered 3 info-gap models of uncertainty:

- **Uniform-bound:**

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- **Envelop-bound:**

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- **Envelop-bound:**

$$\mathcal{U}_{\rm env}(h) = \left\{\phi(x) : \left|\phi(x) - \widetilde{\phi}(x)\right| \le h\psi(x)\right\}, \quad h \ge 0 \tag{57}$$

- **Fourier ellipsoid-bound:**

$$\phi(x) = \sum_{n=n_1}^{n_2} c_n \cos \frac{n\pi x}{L} \tag{58}$$

$$\mathcal{U}_{\rm spec}(h) = \left\{\phi(x) : (c - \widetilde{c})^T W (c - \widetilde{c}) \le h^2\right\}, \quad h \ge 0 \tag{59}$$

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$$\mathcal{U}_{\text{spec}}(h) = \left\{ \phi(x) : \ (c - \widetilde{c})^T W (c - \widetilde{c}) \leq h^2 \right\}, \quad h \geq 0 \tag{63}$$

§ **We now consider the energy-bound info-gap model.**

- $\phi(x)$ **is the load-density function.**
- $\phi(x)$ **usually varies smoothly along the beam.**
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§ **We now consider the energy-bound info-gap model.**

- $\phi(x)$ **is the load-density function.**
- $\phi(x)$ **usually varies smoothly along the beam.**
- $\phi(x)$ **has uncertain strong local deviations from** $\widetilde{\phi}(x)$ **but the total load is bounded:**

$$\mathcal{U}_{\text{energy}}(h) = \left\{\phi(x) : \ \int_0^L \left(\phi(x) - \widetilde{\phi}(x)\right)^2 \le h^2\right\}, \quad h \ge 0 \tag{68}$$

(Energy is a metaphor.)

3 *Conclusion*

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Questions?